Expressions numériques avec radicaux

$\sqrt[n]{x} = x^{1/n}$ $\begin{cases} x \geq 0 \text{ si } n \text{ est pair} \\ x \in \mathbb{R} \text{ sinon} \end{cases}$

$\frac{1}{a^n} = a^{-n}$

$\sqrt[n]{x^m} = x^{m/n}$

$\sqrt[n]{x^n} = \begin{cases} x \text{ si } n \text{ impair} \\ |x| \text{ si } n \text{ pair} \end{cases}$

$\frac{1}{a^n} = \left(\frac{1}{a}\right)^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$(a^n)^m = a^{n\times m} = a^{m\times n} = (a^m)^n$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

① Utiliser des exposants.

$\frac{1}{\sqrt{5}} = \frac{1}{5^{1/2}} = 5^{-1/2}$

$\sqrt[3]{7^2} = 7^{2/3}$

$\sqrt[5]{5^3} = 5^{3/5}$

② Evaluer.

- $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$
- $\sqrt[4]{\frac{1}{16}} = \sqrt[4]{\frac{1}{2^4}} = \sqrt[4]{\left(\frac{1}{2}\right)^4} = \left|\frac{1}{2}\right| = \frac{1}{2}$
- $\frac{\sqrt[5]{-3}}{\sqrt[5]{96}} = \frac{(-3)^{1/5}}{(96)^{1/5}} = \left(\frac{-3}{96}\right)^{1/5} = \left(\frac{-3}{3\times 32}\right)^{1/5}$
  $= \left(-\frac{1}{2^5}\right)^{1/5} = \left(\left(-\frac{1}{2}\right)^5\right)^{1/5} = -\frac{1}{2}.$

③ Utiliser des racines.

- $4^{2/3} = \sqrt[3]{4^2} = \sqrt[3]{(2^2)^2} = \sqrt[3]{2^4}$
  $= \sqrt[3]{2^3 \cdot 2} = \sqrt[3]{2^3} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$
- $11^{-3/2} = \sqrt[2]{11^{-3}} = \sqrt{\frac{1}{11^3}} = \frac{\sqrt[2]{1^2}}{\sqrt{11^2 \times 11}} = \frac{1}{\sqrt[2]{11^2}\sqrt{11}}$
  $= \frac{1}{11\sqrt{11}}.$
- $(-32)^{2/5} = \sqrt[5]{(-32)^2} = \sqrt[5]{(-1)^2(2^5)^2}$
  $= \sqrt[5]{(2^2)^5} = |2^2| = 4.$