$ax^2 + bx + c = 0$ $\quad a \neq 0$

$$a\left[x^2 + \frac{b}{a}x\right] + c = 0$$

$$a\left[\underbrace{x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2} - \left(\frac{b}{2a}\right)^2\right] + c = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - a \cdot \frac{b^2}{4a^2} + c = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$$

- $\Delta > 0$: $\quad a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{\Delta}}{2a}\right)^2\right] = 0$

$$a\left(x + \frac{b}{2a} + \frac{\sqrt{\Delta}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{\Delta}}{2a}\right) = 0$$

$$\Leftrightarrow \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

- $\Delta = 0$ $\quad a\left(x + \frac{b}{2a}\right)^2 = 0 \Leftrightarrow x = -\frac{b}{2a}$

- $\Delta < 0$ $\quad$ pas de solution.

$\Delta = b^2 - 4ac$: discriminant.

- $\Delta > 0$ $\quad ax^2 + bx + c = a(x - x_1)(x - x_2)$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} \qquad x_2 = \frac{-b - \sqrt{\Delta}}{2a}.$$

- $\Delta = 0$ $\quad ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 = a(x - x_0)^2$

$$x_0 = -\frac{b}{2a}.$$

- $\Delta < 0$ $\quad$ pas de solution.

Ex : $\quad x^2 + 2x + 2 = 0$.

$\Delta = b^2 - 4ac = (2)^2 - 4(1)(2) = 4 - 8 = -4 < 0 \longrightarrow$ pas de solution.

Ex : $\quad x^2 + 2x + 1 = 0$.

$\Delta = b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0 \longrightarrow x_0 = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$: une solution.

$x^2 + 2x + 1 = (1)(x + 1)^2$.

Ex : $\quad 3x^2 - 5x - 1 = 0$.

$\Delta = (-5)^2 - 4(3)(-1) = 25 + 12 = 37 > 0$: 2 solutions

$$x_1 = \frac{+5 + \sqrt{37}}{6} \qquad x_2 = \frac{5 - \sqrt{37}}{6}.$$

$3x^2 - 5x - 1 = 3(x - x_1)(x - x_2)$