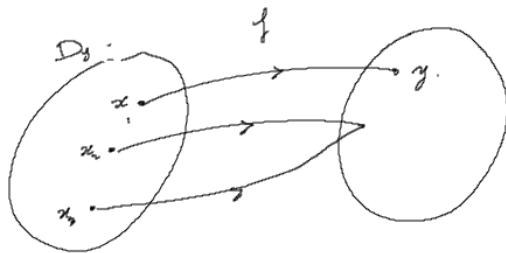


What is a function?

Def: A function f is a rule that assigns for each element $x \in D_f$ a unique element y .



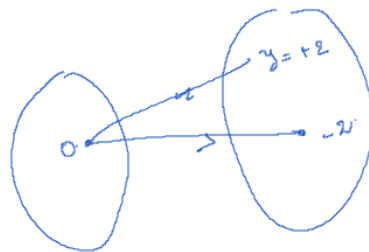
D_f : Domain of definition of f .

$R_f = \{f(x) : x \in D_f\}$
= range of f .

Ex: Is the equation defining a function $y = f(x)$?

i) $y^2 + 3x = 4$.

Choose $x=0$: $y^2 = 4 \Leftrightarrow y = \pm 2$.
No, the equation doesn't define a function because for $x=0$, we assign two values of y .



Ex: $f(x) = \sqrt{x} + 3$.

For any $x \in [0, +\infty)$, we assign the unique value $y = \sqrt{x} + 3$.

So f is a function.

ii) $x^2 y + 9y = 5 \Leftrightarrow y(x^2 + 9) = 5 \Leftrightarrow y = \frac{5}{x^2 + 9}$
For each $x \in \mathbb{R}$, $\frac{5}{x^2 + 9}$ is well defined and assigned to x .

This is a function.

iii) $3x + 6|y| = 0 \Leftrightarrow x + 2|y| = 0 \Leftrightarrow |y| = -\frac{x}{2}$.
Choose $x = -2$, then $|y| = 1 \Leftrightarrow y = \pm 1$. $(-2, 1), (-2, -1)$ satisfy the equation with the same x .

- $f(0) = \sqrt{0} + 3 = 3$.
- $f(9) = \sqrt{9} + 3 = 3 + 3 = 6$.
- $f(-2)$ is not defined.
- $f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{4}} + 3 = \frac{1}{2} + 3 = \frac{7}{2}$.
- $f(x^2 + 1) = \sqrt{x^2 + 1} + 3$.
- $f(\sqrt{x} + 1) = \sqrt{\sqrt{x} + 1} + 3$.