Radical Exponents

n^{th} root of x:

- n odd: $x^{1/n} = y \iff x = y^n$ $(\forall x \in \mathbb{R})$
- n even $y = x^{1/n}$ for $x \geq 0$.
- $\sqrt[n]{x} = x^{1/n}$
- $x^{m/n} = \sqrt[n]{x^m} = \left(x^{1/n}\right)^m$.
- $x^{-m/n} = \frac{1}{x^{m/n}}$

$(ab)^n = a^n \cdot b^n$.

$(-1)^{48} = (-1)^{2\times 24} = \left((-1)^2\right)^{24}$

$= 1^{24} = 1$

$\left(a^n\right)^m = a^{n \cdot m}$ | $\frac{1}{a^{-n}} = a^n$

(1) Simplify:

$\sqrt[11]{x^{19}} = x^{19/11}$

$t^{1/4} \cdot t^{-8} = t^{\frac{1}{4}-8} = t^{\frac{1-32}{4}} = t^{-31/4}$

$(-3y^{-1}) \cdot (4y^{-4/3}) = (-3)(4)\, y^{-1} \cdot y^{-4/3} = -12\, y^{-1-4/3} = -12\, y^{-7/3}$

(2) Eliminate the negative power.

$$\left(\frac{-x^{1/3}}{y^{1/8} z^{-1/3}}\right)^{48} = \frac{\left(-x^{1/3}\right)^{48}}{\left(y^{1/8} z^{-1/3}\right)^{48}} = \frac{(-1)^{48}\left(x^{1/3}\right)^{48}}{\left(y^{1/8}\right)^{48}\left(z^{-1/3}\right)^{48}}$$

$$= \frac{x^{\frac{48}{3}}}{y^{\frac{48}{8}} z^{-\frac{48}{3}}}$$

$$= \frac{x^{48/3}}{y^6 z^{-16}} = \frac{x^{48/3} \cdot z^{16}}{y^6}$$