$\boxed{\text{Radical Expressions}}$

Conjugate of $a+b$ is $a-b$.

(used for rationalizing an expression with a radical).

$$\sqrt[n]{a} = a^{1/n} \qquad \sqrt[n]{\sqrt[m]{a}} = \left(a^{1/m}\right)^{1/n} = a^{\frac{1}{mn}} = \sqrt[nm]{a}.$$

$$\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n}\cdot b^{1/n} = \sqrt[n]{a}\cdot\sqrt[n]{b}.$$

$$\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/n} = \frac{a^{1/n}}{b^{1/n}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

Ex: Rationalize the numerator:

- $\sqrt{\frac{17}{x}} = \frac{\sqrt{17}}{\sqrt{x}} = \frac{\sqrt{17}\cdot\sqrt{17}}{\sqrt{x}\cdot\sqrt{17}} = \frac{17}{\sqrt{17x}}.$

- $\sqrt{x^2+31} - x = \frac{\left(\sqrt{x^2+31}-x\right)\left(\sqrt{x^2+31}+x\right)}{1\left(\sqrt{x^2+31}+x\right)}$

$$= \frac{\left(\sqrt{x^2+31}\right)^2 - x^2}{\sqrt{x^2+31}+x} = \frac{(x^2+31)-x^2}{\sqrt{x^2+31}+x} = \frac{x^2}{\sqrt{x^2+31}+x}.$$

$$\left(\sqrt{x^2+31}\right)^2 = \left[(x^2+31)^{1/2}\right]^2 = (x^2+31)^{\frac{1}{2}\times 2} = (x^2+31)^1 = x^2+31.$$

Ex. Rationalize the denominator

- $-2 + \frac{x-7}{6(\sqrt{x}-\sqrt{7})} = -2 + \frac{(x-7)(\sqrt{x}+\sqrt{7})}{6(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})}$

$$= -2 + \frac{\cancel{(x-7)}(\sqrt{x}+\sqrt{7})}{6\,\cancel{(x-7)}} = -2 + \frac{\sqrt{x}+\sqrt{7}}{6}$$

- $\frac{x^2 - \overbrace{256}^{16^2} y^2}{\sqrt{x} - 4\sqrt{y}} = \frac{(x-16y)(x+16y)(\sqrt{x}+4\sqrt{y})}{(\sqrt{x}-4\sqrt{y})(\sqrt{x}+4\sqrt{y})}$

$$= \frac{(x-16y)(x+16y)(\sqrt{x}+4\sqrt{y})}{(\sqrt{x})^2 - (4)^2(\sqrt{y})^2}$$

$$= \frac{\cancel{(x-16y)}(x+16y)(\sqrt{x}+4\sqrt{y})}{\cancel{x-16y}}.$$