

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

$$\text{Set: } \Delta = b^2 - 4ac : \text{ discriminant}$$

$$a \left[ x^2 + \frac{b}{a}x \right] + c = 0$$

$$a \left[ x^2 + 2 \left( \frac{b}{2a} \right) x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 \right] + c = 0$$

$$a \left[ \left( x + \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 \right] + c = 0$$

$$a \left( x + \frac{b}{2a} \right)^2 - a \left( \frac{b^2}{4a^2} \right) + c = 0$$

$$a \left( x + \frac{b}{2a} \right)^2 = a \frac{b^2}{4a^2} - c$$

$$\left( x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2} = \frac{\Delta}{4a^2}$$

$$* \Delta > 0 : x + \frac{b}{2a} = \pm \frac{\sqrt{\Delta}}{2a} \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$* \Delta < 0 : \text{no solution}$$

$$* \Delta = 0$$

$$x = \frac{-b}{2a}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$ax^2 + bx + c = a \left( x + \frac{b}{2a} \right)^2$$

$$\text{Ex: } x^2 + 2x + 1 = 0$$

$$a=1 \quad b=2 \quad c=1$$

$$\Delta = b^2 - 4ac = 2^2 - 4(1)(1) = 0 \rightarrow x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

$$\text{solution: } x = -1$$

$$x^2 + 2x + 1 = (x+1)^2$$

$$\text{Ex: } x^2 + 2x + 2 = 0$$

$$\Delta = b^2 - 4ac = (2)^2 - 4(1)(2) = 4 - 8 = -4 < 0$$

no solution.

$$\text{Ex: } 3x^2 - 5x - 1 = 0$$

$$\Delta = b^2 - 4ac = (-5)^2 - 4(3)(-1) = 25 + 12 = 37 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{5 - \sqrt{37}}{6}$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{5 + \sqrt{37}}{6}$$

solutions.

$$3x^2 - 5x - 1 = 3(x - x_1)(x - x_2)$$