

Parabola.

The graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola.

$$f(x) = a(x-h)^2 + k \quad ; \quad (h, k) : \text{vertex.}$$

Ex 1: $f(x) = -x^2 + 6x - 12$

$$\begin{aligned} &= -[x^2 - 6x] - 12 \\ &= -[x^2 - 2 \times 3x + 3^2 - 3^2] - 12 \\ &= -[(x-3)^2 - 9] - 12 \\ &= -(x-3)^2 + 9 - 12 \\ &= -(x-3)^2 - 3. \end{aligned}$$

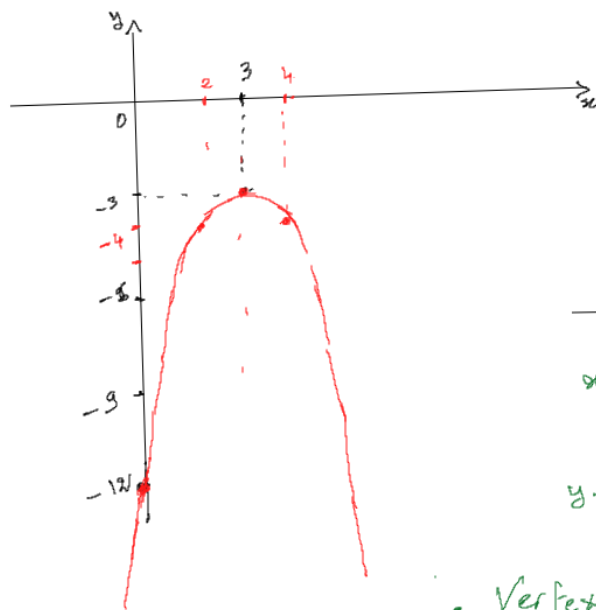
Vertex = $(3, -3)$

y-intercept = $(0, -12)$

x-intercepts: $y=0 \Rightarrow (x-3)^2 + 3 = 0$
no solution.

Two more points: $(4, -4), (2, -4)$

$\max_{\mathbb{R}} f = -3 = f(3) \quad ; \quad \min_{\mathbb{R}} f = \text{DNE}$



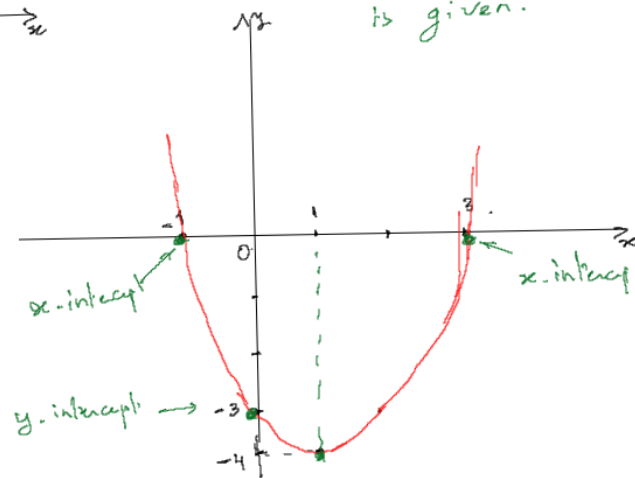
• Equation

$$y = a(x-h)^2 + (-4)$$

• With $x=0, y=-3=a-4$
then $a=4-3=1$.

$$y = (x-1)^2 - 4$$

Ex 2: The graph of a parabola is given.



• Vertex: $(1, -4)$

• y-intercept: $(0, -3)$

• x-intercepts: $(-1, 0), (3, 0)$

• $\max_{\mathbb{R}} f = \text{DNE}$

$\min_{\mathbb{R}} f = -4 = f(1)$