

Nonlinear equations

- Steps:
- Factor the expression into simple terms: $ax+b$.
 - Table of signs:

Ex: Solve

$$\frac{2x-1}{x+5} \geq 0$$

x	-7	-5	$\frac{1}{2}$	$+\infty$
$2x-1$	-	-	0	+
$x+5$	-	+	+	+
$\frac{2x-1}{x+5}$	+	-	0	+

$$\begin{aligned} & \bullet 2x-1 > 0 \\ & \Leftrightarrow 2x > 1 \\ & \Leftrightarrow x > \frac{1}{2} \end{aligned}$$

$$\begin{aligned} & \bullet x+5 > 0 \\ & \Leftrightarrow x > -5 \end{aligned}$$

$$\bullet \frac{2x-1}{x+5} \geq 0 \Leftrightarrow x \in (-7, -5) \cup [\frac{1}{2}, +\infty)$$

$$\bullet \frac{2x-1}{x+5} < 0 \Leftrightarrow x \in (-5, \frac{1}{2})$$

Ex. Solve: $F(x) = \frac{(2x-1)^{\frac{1}{3}}(-4x+1)^2}{(x+5)^3(x^2+7)} \geq 0$

$$\bullet (-4x+1)^2 \geq 0 \text{ and } (-4x+1)^2 = 0 \Leftrightarrow x = \frac{1}{4}$$

$$\bullet (x^2+7) > 0 \text{ for any } x \in \mathbb{R}.$$

$$\bullet (2x-1)^{\frac{1}{3}} > 0 \Leftrightarrow x > \frac{1}{2}$$

$$\bullet (x+5)^3 = \underbrace{(x+5)^2}_{>0} (x+5) \text{ for any } x \neq -5.$$

$$\frac{(2x-1)^{\frac{1}{3}}(-4x+1)^2}{(x+5)^3(x^2+7)} = \frac{(2x-1)^{\frac{1}{3}}}{(x+5)} \cdot \frac{(-4x+1)^2}{(x+5)^2(x^2+7)} \geq 0$$

x	-5	$\frac{1}{4}$	$\frac{1}{2}$	$+\infty$
$(2x-1)^{\frac{1}{3}}$	-	-	0	+
$x+5$	-	+	+	+
$F(x)$	+	-	0	+

Solutions for $F(x) \geq 0$ are: $(-7, -5) \cup \{\frac{1}{4}\} \cup [\frac{1}{2}, +\infty)$