Factoring a polynomial

- Find common factors in the given expression.
- $(a^2 - b^2) = (a-b)(a+b)$

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(1) $x^3 - 27$

$= x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

Solve: $x^2 + 3x + 9 = 0$.

$\Delta = 3^2 - 4(9) < 0 \longrightarrow$ no roots.

(2) $4x^2 - 4x + 1$

- 1st method: $(2x)^2 - 2(2x)(1) + 1^2 = (2x-1)^2$
- 2nd method: solve $4x^2 - 4x + 1 = 0$.

$\Delta = (-4)^2 - 4(4)(1) = 0 \longrightarrow x_0 = \frac{4}{2(4)} = \frac{1}{2}$.

$4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2 = 4\left(\frac{2x-1}{2}\right)^2 = (2x-1)^2$

(3) Find the common factor between.

$y^2(y-1)^4$ and $5y^6(y-1)^2$.

Ans: $y^2(y-1)^2$.

(4) Factor $3x^3 + 5x^2 - 6x - 10$.

$= x^2(3x+5) - 2(3x+5)$.

$= (3x+5)(x^2 - 2)$.

$= (3x+5)(x^2 - (\sqrt{2})^2)$.

$= (3x+5)(x+\sqrt{2})(x-\sqrt{2})$

(5) Factor $2(x+8)^2 + 5(x+8) - 3$.

Set $t = x+8$. Then $2t^2 + 5t - 3$

$\Delta = 5^2 - 4(2)(-3) = 49 = 7^2$

$t_1 = \frac{-5+7}{2} = 1$

$t_2 = \frac{-5-7}{2} = -6$.

$2t^2 + 5t - 3$

$= 2(t-1)(t+6)$.

$= 2(x+8-1)(x+8+6)$

$= 2(x+7)(x+14)$