Domaine of Definition

$D_f = \{ x \in \mathbb{R} : f(x) \in \mathbb{R} \}$.

$f(x) = P(x)$ $\quad D_f = \mathbb{R}$.

$f(x) = \dfrac{P(x)}{Q(x)}$ $\quad D_f = \mathbb{R} \setminus \{ x : Q(x) = 0 \}$.

$f(x) = \sqrt{P(x)}$; $D_f = \{ x : P(x) \geq 0 \}$.

① $f(x) = x^2 - 3x + 2$ $\quad D_f = \mathbb{R}$.

② $f(x) = \dfrac{1}{x^2 - 3x + 2}$ $\quad D_f = \{ x \in \mathbb{R} : x^2 - 3x + 2 \neq 0 \}$.

Solve: $x^2 - 3x + 2 = 0$: $\Delta = b^2 - 4ac = (-3)^2 - 4(1)(2) = 1 > 0$

$x_1 = \dfrac{-b + \sqrt{\Delta}}{2a} = \dfrac{3+1}{2} = 2$; $x_2 = \dfrac{-b - \sqrt{\Delta}}{2a} = \dfrac{3-1}{2} = 1$.

$\left(ax^2 + bx + c = 0 \Rightarrow a(x - x_1)(x - x_2) ; \Delta > 0 \right)$.

$D_f = \mathbb{R} \setminus \{1, 2\}$

$= (-\infty, 1) \cup (1, 2) \cup (2, +\infty)$

③ $f(x) = \sqrt{x^2 - 3x + 2}$

$D_f = \{ x \in \mathbb{R} : x^2 - 3x + 2 \geq 0 \}$.

x	$-\infty$		1		2		$+\infty$
$x - 1$		$-$	0	$+$		$+$	
$x - 2$		$-$		$-$	0	$+$	
$x^2 - 3x + 2$		$+$	0	$-$	0	$+$	

$x^2 - 3x + 2 = (x - 1)(x - 2)$

• solve $x - 1 \geq 0 \iff x \geq 1$.

$D_g = (-\infty, 1] \cup [2, +\infty)$.

④ $f(x) = \dfrac{1}{x^2 - 3x + 2} + \sqrt{x^2 - 3x + 2}$

$D_f = \{ x \in \mathbb{R} : x^2 - 3x + 2 \neq 0 \text{ and } x^2 - 3x + 2 \geq 0 \} = (-\infty, 1) \cup (2, +\infty)$