

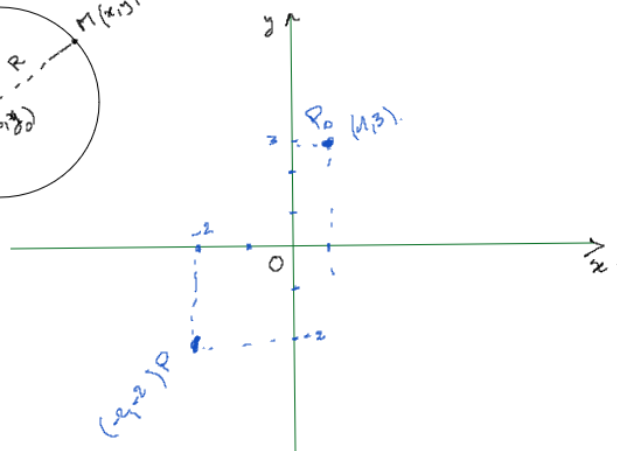
Circle.

$$\text{Circle} = \{ M(x,y) : OM = R \}.$$

$$OM = \sqrt{(x-x_0)^2 + (y-y_0)^2} = R.$$

$$OM^2 = (x-x_0)^2 + (y-y_0)^2 = R^2$$

equation of the circle
centered at (x_0, y_0) , with
radius R .



Ex: Find the equation of the circle
centered at $(1, 3)$ and passing
through $(-2, -2)$.

$$\begin{aligned} \text{* The radius: } P_0P &= \sqrt{(-2-1)^2 + (-2-3)^2} \\ &= \sqrt{9+25} = \sqrt{34}. \end{aligned}$$

$$R = \sqrt{34}.$$

* Equation of the circle:

$$(x-1)^2 + (y-3)^2 = 34.$$

Ex: i) $x^2 + y^2 - 12x + 2y = -36$

ii) $x^2 + y^2 - 12x + 2y = -37$

iii) $x^2 + y^2 - 12x + 2y = -38$

Identify whether it is an equation
of a circle.

$$\begin{aligned} \text{i) } x^2 - 12x + y^2 + 2y &= -36. \\ (x-6)^2 - 36 + (y+1)^2 - 1 &= -36. \\ (x-6)^2 + (y+1)^2 &= 37 - 36 = 1 \\ (x-6)^2 + (y-(-1))^2 &= 1^2. \end{aligned}$$

Circle with center $(6, -1)$
radius $R=1$.

$$\text{ii) } \underbrace{(x-6)^2 + (y+1)^2}_{\geq 0} = 37 - 37 = 0$$

$$\Leftrightarrow (x,y) = (6, -1).$$

only one point.

$$\text{iii) } \underbrace{(x-6)^2 + (y+1)^2}_{\geq 0} = 37 - 38 = \underbrace{-1}_{< 0}$$

no points: $\emptyset$.