## **Exercises: Section 1.1 Functions**

- 1) Complete the following sentences:
  - a) Set is
  - b) Function is
  - c) f(x) means
  - d) Domain of function is
  - e) Range of a function is
  - **f)** Independent variable is
  - g) Dependent variable is
- 2) At a coffee shop, the menu consists of items and their prices. Is price a function of the item? Is the item a function of the price? Justify your answers.
- 3) At Ontario Tech University, the final grade a student earns in the course corresponds to a letter grade (see <u>Ontario Tech Grading</u>). Is the letter grade a function of the percentage a student can obtain in the course? Is the percentage a student can obtain in the course a function of the letter grade? Justify.
- 4) Consider information related to bank accounts.
  - a) Is the account balance a function of the bank account number? Justify.
  - **b)** Is the bank account number a function of the account balance? Justify.
- 5) Introduce function notation of your choice using variable names to represent a function that takes as input the name of a month, and gives as output the number of days in that month. State clearly what each variable represents.
- A function N(y) gives the number of police officers, N, in a town in year y. What does
  N(2005) = 300 tell us?
- 7) Which of these tables define a function (if any)? Justify.

a)

Input	Output
2	1
5	3
8	6

Input	Output
-3	5
0	1
4	5

b)		
	Input	Output
	1	0
	5	2
	5	4

8) Describe the relationship shown in the graph below as a function and describe its domain and range in interval form.



9) Using the table shown for Q(n):

n	1	2	3	4	5
Q	8	6	7	6	8

- a) Evaluate Q(3) and interpret your result (describe in words what the answer means).
- b) Solve Q(n) = 6 and interpret your result.
- c) Evaluate Q(7). Interpret your result.
- d) Solve Q(n) = 1. Interpret your result.

**10)** Which of these graphs defines a function? Justify your answers.



**12)** For the function y(x) with the following graph:

- a) State the value of y(1).
- b) Estimate the value of y(-5/2).
- c) Estimate the value(s) of x such that y(x) = 3.
- d) For what values is y(x) = 7?
- e) State the domain and the range of y(x).
- F) Determine the interval(s) on which the function is decreasing, increasing and where it is constant.

**13)** Given the function  $k(t) = t^3 + 2$ .

- a) Evaluate k(2).
- **b)** Solve k(t) = 1.

**14)** Given the function  $h(p) = p^2 + 2p$ ,

- a) Evaluate h(4).
- **b)** Solve h(p) = 3.
- **15)** Given the function  $g(m) = \sqrt{m-4}$ 
  - a) Evaluate g(5)
  - **b)** Solve g(m) = 2

**16)** If 
$$f(x) = \frac{x^2 + 1}{x\sqrt{-2x + 10}}$$
, find

- **a)** f(3)
- **b)** f(-3)
- **c)** f(a-3)

**17)** Given  $f(x) = \frac{2x-1}{x^2+2}$ , find

- **a)** f(3)
- **b)** f(-3)
- c) f(x+2)
- **d)** f(x+h)

**18)** If possible, express the relationship 2n + 6p = 12 as a function p(n).

**19)** Consider the relationship  $3m^2 - 15n = 7$ . Use this relationship to express the following functions, if possible. If this is not possible, justify why not.

- a) Express *n* as a function of *m*.
- b) Express *m* as a function of *n*.



**20)** Describe the domain of each of the following functions using interval notation:

- a)  $f(x) = 5x^4 3x + 1$ b)  $g(x) = \frac{3x+5}{x^2+4x-5}$ c)  $t(m) = \frac{\sqrt{2m-3}}{\sqrt{5-m}}$ d)  $f(t) = \frac{t}{\sqrt[3]{t+3}}$ e)  $k(x) = \frac{x}{e^{x}-1}$ f)  $p(g) = 3 + \frac{4\sqrt[5]{5-7g}}{a^2+4}$
- 21) A group of network technicians tested an online software and determined a model that describes a relationship between the percentage P(t) of registered users using the software over time t during the course of 24 hours. The relationship is presented by the graph below.
  - a) How can you tell that the graph represents a function?
  - b) Describe the domain and the range of *P*.
  - c) What is the independent variable and what is the dependent variable in this relationship?
  - d) At what time is the percentage of software users highest?



- e) Describe what you observe from the graph about the relationship between the time of day and the percentage of registered users using the software. Be as specific as possible, use numbers (your best estimates) and appropriate terminology and form in your description.
- 22) A data storage company rents server space for a flat annual fee of \$150 and storage charge of \$0.39 per GB for less than 1.8 petabytes per annual contract (one petabyte is 1000 terabytes and one terabyte is 1000 GB).
  - a) Write the formula for the total cost *C* to rent server space as a function of the gigabytes of data storage requirement *s*.
  - b) What are the domain and the range of this function?
  - c) Find the total cost to rent server space for 3.5 terabytes of storage space.
  - d) Determine how much server space was rented under the annual contract if the bill was \$1203.
- **23)** A phone data plan has a basic charge of \$30 a month. The plan includes first 2GB free and charges \$10 for each additional GB, up to 8 GB total usage, after which it charges \$15 for each additional GB. If *d* is the amount of data used (in GB) and *C* is the total monthly cost:
  - a) Express C(d) as a (piece-wise) formula.
  - **b)** Identify the independent and the dependent variables of *C*.
  - c) Identify the domain and the range of *C*.
  - d) Graph C as a function of d for  $0 \le d \le 10$ .
  - e) Calculate the cost if 9GB were used.

24) In making strategic decisions about product pricing, we study the relationship between market demand for an item and the price of the item. Suppose the market demand for *x* number of units and the price of the unit *p* of a particular product satisfy the following relationship:

$$9x + 150p = 6500$$

- a) If possible, express the demand as a function of the price and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- b) If possible, express the price as a function of the demand and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- **25)** If the revenues *R* from sales of *x* units of a certain product and the cost *C* of manufacturing *x* units of the same product can be expressed as

$$R(x) = 18x$$
 and  $C(x) = 9.37x + 25000$ 

- a) Determine the profit function *P* in terms of the number of units *x*.
- b) Calculate *P*(2000) and interpret what that means.
- c) Calculate P(4000) and interpret what that means.
- d) How many units would they have to produce and sell to break even?
- e) How many units would they have to produce and sell to earn \$20,000 in profit?
- **26)** A closed rectangular box with volume 6 m<sup>3</sup> has length three times the width. Express the surface area of the box as a function of the width of the box.
- **27)** An open rectangular box with volume 8 ft<sup>3</sup> has length twice the width. Express the height of the box as a function of the width.
- **28)** Suppose a box whose length of the base is twice the width and volume of 120 cm<sup>2</sup> is made of material that costs \$10/cm<sup>2</sup>. Express the cost of the material as a function of the width of the box.
- **29)** A cylindrical can is made to hold 1 L of oil. Write the surface area of the can:
  - a) as a function of the radius of its base.
  - **b)** as a function of its height.
- **30)** If 1200 cm<sup>2</sup> of material is available to make a box with a square base and an open top, write the volume of the box as a function of the width of its base.