

Physics 1D03 & 1E03

Physics 1D03: Introductory Mechanics

Physics 1E03: Waves, Electricity, and  
Magnetic Fields

Laboratory Manual – Reference Material

Last updated: September 2021


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# 1 Laboratory Objectives

Please ensure that you carefully read and understand the sections of the course outline pertaining to laboratory experiments. You must attend and complete all the labs during the regular term to receive credit; there is no opportunity to do the labs after the end of term or during the summer. If you miss a lab over the course of the term, you will be given the opportunity to complete it as a make-up lab at the end of the term.

The laboratory component of Physics 1D03/1E03 has objectives beyond illustrating the concepts treated in the lectures. It is intended to:

1. Teach measurement techniques, including
  - (a) selection and manipulation of measuring instruments,
  - (b) understanding the way in which instruments work and their limitations,
  - (c) assessment and understanding of the uncertainty in measurements,
  - (d) using preliminary measurements to design strategies for making further accurate measurements.
2. Introduce the student to the analysis of experimental data, including
  - (a) use of numerical and graphical methods,
  - (b) simple data analysis through Microsoft Excel,
  - (c) treating the propagation of experimental uncertainties in calculations.
3. Give the student practice in the written presentation of results, including
  - (a) using graphical methods to display the results of measurements,
  - (b) assessing the value of the information obtained and drawing reasonable conclusions,
  - (c) presenting clear and precise reports through Microsoft Word.
4. Illustrate physical phenomena (not necessarily limited to those treated in the lectures).

## 2 Safety in the Laboratory

Safety must be your first concern in a laboratory, as a student or as an employee. You must be aware in advance of potential hazards, and have the training and personal protective equipment (PPE) needed to work safely. A lab supervisor is responsible for ensuring that students and employees are properly informed and trained, but you should also actively seek clarification if you are uncertain about hazards and safe practices.

### 2.1 Physics 1D03/1E03 Labs

The teaching assistants in the lab, and the lab coordinator in BSB-B117 will demonstrate how to use the equipment at the beginning of each lab session, and provide guidance during the lab. Ask them if you have any questions or concerns. If any of the equipment appears damaged or unsafe, for example, a loose prong on an electrical plug, then you should consult the TA or the lab coordinator before attempting to use it.

#### General rules

- Food and drinks are not permitted in the lab room;
- Personal electronic devices, such as smartphones and tablets, are not permitted to be used in the lab;
- Shoes or boots with closed toes (no sandals) must be worn in the lab;
- Long pants are preferred, if possible;
- Lab coats and protective glasses are not necessary.

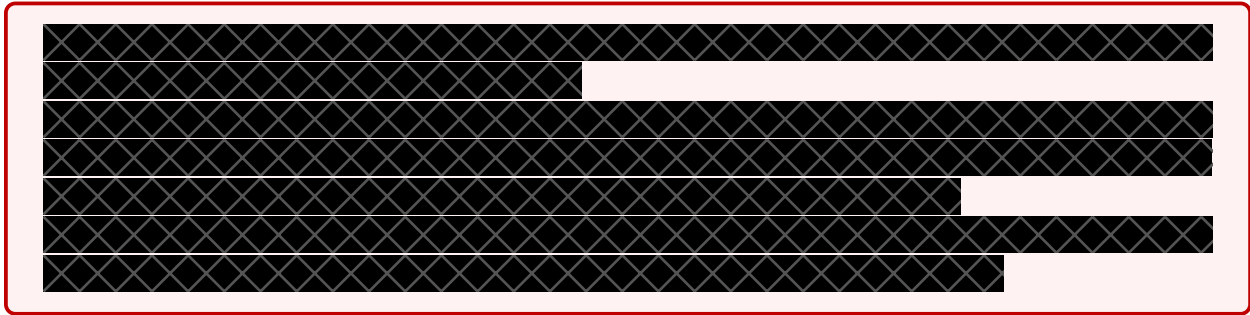
#### Potential hazards

**Spills and splashes:** You will not work with any hazardous substances in the Year I Physics Labs, and protective clothing is not required.

**Falling objects:** None of the equipment is particularly heavy, but you should pay attention when working to avoid dropping it onto the floor (or your foot).

**Electrical equipment:** In the Physics 1E03 labs, you will use batteries and low-power electrical power supplies (15 volts, 20 mA). These voltages and currents do not present a hazard.

These are general rules. You should read the instructions for each lab a day or two before, and check for any additions or modifications specific to that lab.



### 3 Laboratory Procedures and Reports

#### 3.1 General lab guidelines

Labs are carried out in groups of 2. If you have no prearranged lab partner, please find one after arriving at the lab room. Measurements are made by the partners together during the lab period, and recorded onto the MS Word report on the computers.

**All lab reports are to be completed during the lab period and submitted before you leave.** You and your lab partner should work together in gathering and analyzing data to produce a joint lab report. It is NOT acceptable for one partner to do the set-up and measuring while the other does the calculations. All components of the lab should be shared and understood by both partners, and both must stay until the report is submitted to the TA. You must finish within the three hours allocated to the lab, so you will have to be well prepared when you arrive. Read the lab instructions at least a day in advance, and complete the brief pre-lab worksheet. EACH student must hand in the pre-lab worksheet on entering the lab. The TA will leave after the three-hour lab period has expired; whatever you have written by then constitutes your lab report and will determine your mark.

**You must be on time.** If you are late, you may be asked to leave, and do the lab at a later date (it is up to you to make the arrangements). **You should bring only your pre-lab worksheet and your lab manual. Phones, computers and tablets are not allowed during the lab; turn them off and put them away out of sight.** Bringing a completed lab report or similar material “for reference”, or referring to it while working, will be treated as academic dishonesty. Any indication that your electronic devices such as phones, computers, or tablets are being used to communicate outside of the lab will be subjected to inquiry and may be considered academic dishonesty.

#### 3.2 Lab report structure

Lab reports are completed by filling out the provided templates with the data, graphs and results obtained during the lab. No additional preamble (“Purpose”, “Method”, etc.) is needed. All data tables must be filled in (unless a section is indicated as optional), including notation of uncertainties and units. All graphs (graphs will be solely generated through Microsoft Excel) must be included with the report, with titles and axis labels. You are

encouraged to re-read the "Uncertainty Analysis" and the "Excel Basics" section of this lab manual before each lab.

The experimental procedure for each lab contains questions. Some of the questions ask you to decide on the details of how you will perform a measurement. These should be answered in the order in which they appear in the manual. Other questions are to guide you in discussing your results and drawing conclusions. For most questions, a sentence or two of explanation is expected. A one-word "Yes." or "No." answer, with no justification, is inadequate. Avoid vague, unsupported generalizations in your answers to questions. Where appropriate, include remarks about your measurement procedure, such as methods you adopted to minimize the uncertainty of your results. Tie your answer to your results: where appropriate, quote your numerical results and/or uncertainties in your answers.

Your lab report should include your name, student number and lab section; and the name, student number, and lab section of your partner. If you are doing the lab with a different section (only by prior arrangement with the TA in charge), keep a careful record of the date, location, and the name of the TA present in case you need to communicate them to the head TA or your usual TA. In all cases, make sure you sign the attendance sheet. Lab reports will be marked according to the attached grading rubric. The marker will be looking for the following:

1. *Experimental technique*: The quality and completeness of the data and thoughtful answers to questions about the procedure.
2. *Data analysis*: Tables, graphs, and uncertainty calculations. Are the graphs complete and properly labelled? Are explanations for uncertainties demonstrated in the questions. Are appropriate sample calculations shown to the TA's during the lab? Is the analysis correct?
3. *Discussions and conclusion*: Answers to questions in the manual are expected in the report. You must support your answers and conclusions with references to specific features of your data, as shown in the tables and graphs. Vague statements (e.g., "The causes might be outside influences or human error") have no value. Show that you have thought about your results.
4. *Presentation*: Overall organization, clarity and legibility.

Keep in mind that you may only be awarded marks for material contained within your lab report and presented clearly. If an answer is ambiguous or incomplete, you may not be awarded full marks for it. A standard procedure will be put in place to handle lab grading issues. The specific steps to follow will be communicated to you after each set of lab grades is released. If you believe there are issues with how your lab was graded, you will be able to request a regrading. Please note that such a request will result in a full regrading of your lab, and may thus in some cases lead to an adjusted grade lower than your original grade.

### 3.3 Make-up labs

You may miss up to one lab over the course of the semester and complete it as a make up at the end of the term (see Mosaic for exact dates). During the make-up lab period,

students may complete any one of the four labs. In order to arrange for a make-up lab during this period, send an e-mail one week before the make-up week to Dr. Buntar at buntarv@mcmaster.ca. In your e-mail provide the following information:

1. Your name
2. Number of the lab you have missed and need to make-up
3. Day you would like to do the lab

You may also have the opportunity to make-up a lab with another group before the make-up week while this experiment is still running (space permitting). To arrange for that, e-mail Dr. Buntar with an explanation why the lab has been missed and the day and time (8:30 a.m. or 2:30 p.m.) you would like to do the lab. When the date is arranged, come few minutes before the lab starts to Dr. Buntar's office (BSB B117) to be ushered to a laboratory.

You may miss and make-up another lab over the course of the semester if you have submitted a McMaster Student Absence Form (MSAF) for it. You would then follow the procedure outline above to arrange for make-up.

**Note: For every lab you miss, you will need an MSAF in order to be allowed to do the make-up lab.**

## 4 Uncertainty Analysis

A quantity measured as the result of an experiment is meaningless unless we have an estimate of the uncertainty associated with it. For example, imagine you are a forensic scientist in charge of investigating the untimely death of Colonel Mustard, found dead in the Thode Library this morning. Your best estimate is that he was murdered at 9:45 pm last night. If Miss Scarlett was seen leaving The Phoenix at exactly 10 pm, you need an estimate of the uncertainty on Colonel Mustard's time of death to rule her out as a suspect. If you are confident your estimate is correct "give or take" 5 minutes, then Miss Scarlett is innocent. If however, your estimate is only correct "give or take" 30 minutes, then Miss Scarlett should remain a suspect.

Although calculating uncertainties may seem tedious at times, it is an essential scientific skill. Handling experimental uncertainties is one of the most widely applicable skills you will learn during these experiments.

There are many textbooks that explain how to deal with experimental uncertainties. To write this section we have used the two following textbooks:

- *Data Reduction and Error Analysis for the Physical Sciences*, P.R. Bevington and D. K. Robinson, McGraw Hill.
- *The Uncertainty in Physical Measurements*, P. Fornasini, Springer.

For the experiments in this course, it is only necessary to be familiar with the rules outlined in this document. However, you may find it useful to do some additional reading about uncertainties.

## 4.1 Sources of uncertainties

When you measure a quantity (e.g. the length of a table, the mass of a ball, the time it takes an object to fall a set distance under the influence of gravity, etc.) the value that you obtain may not be accurate. This means your measurement may differ from the true value of that quantity, and this can be due to a number of reasons:

- You may have made a mistake in your measurement or when reporting that measurement. These types of uncertainties are called *illegitimate errors* and should be avoided by carefully performing and repeating experiments, by having your lab partners check you when you are performing a measurement, and by taking careful notes.
- The instrument you are using may not be properly calibrated, or you may be introducing a bias in your measurement because of the way you are reading the instrument. For example, if you do not take into account the presence of a meniscus when reading the level of a liquid, or if you do not take into account your reaction time when measuring the duration of an event. These types of uncertainties are called *systematic uncertainties*, and when they cannot be avoided, their presence should be reported and taken into account.
- The precision of the instrument you are using for your measurement, and the precision of your reading of this instrument, will always be limited, introducing *random uncertainties* in your measurements. These uncertainties should be limited as much as possible, and the remaining uncertainty should be estimated (e.g. by taking into account the precision of your measuring instrument, often given by the manufacturer).

## 4.2 Calculating experimental uncertainties

There are two ways you can estimate the uncertainties associated with a measurement.

1. **Direct evaluation of the experimental uncertainty.** If you know the precision of the instrument you are using, and if you think you are using the instrument optimally (no usage or reading uncertainties), then your experimental uncertainty should be equal to this precision. For example, if you are using a balance with a maximum precision of 1 g, then the uncertainty associated with the mass you are measuring should be  $\pm 1$  g.
2. **Statistical evaluation of the experimental uncertainty.** If your experimental uncertainty is random (i.e. not systematic), then repeating your measurement several times will result in several slightly different values. The average of those values should be close to the true value. To estimate the uncertainty, or how close the average is to the true value, you can calculate the standard deviation of your list of measurements. If you have a large number of values (more than about 5), then you should use the standard error of the mean to estimate your uncertainty. This is calculated as the standard deviation divided by the square root of the number of trials.

You should always use both of these methods to check which one gives you the largest uncertainty and report that with your measurement. You always want to report the larger



uncertainty that you estimate to be conservative about the strength of your experiment. Underestimating uncertainties can lead to incorrect conclusions. In the Colonel Mustard example, if you underestimate the uncertainty associated with the time of death you measured, you might wrongly rule out Miss Scarlet as a suspect. On the other hand, if you overestimate your uncertainty, then you are admitting you cannot make a decision without gathering more information. This is a fundamental strength of the scientific method, not a weakness. Identifying when insufficient data has been observed to draw a concrete conclusion helps to prevent guessing.

When discussing uncertainties, be aware that there is a difference between the terms “accuracy” and “precision.” Accuracy refers to how close your measurement of a quantity is to the true value of that quantity, and it is affected by illegitimate, systematic, and random uncertainties. Precision refers to how reproducible your measurements are, or in other words, the spread in your measurement values. Precision is not affected by illegitimate or systematic uncertainties, only by random uncertainties.

As you can see, there are several methods of calculating uncertainties. Sometimes, there is no right or wrong way, just two (or more) different ones. What we would like you to do is:

1. Try to estimate your experimental uncertainty.
2. Explain how you have estimated your uncertainty.
3. When multiple options appear valid, choose the method that gives you the largest uncertainty.

### 4.3 Propagating uncertainties

When combining measurements with uncertainties, the result will also have an uncertainty. For example, to calculate the momentum,  $p$ , of a thrown ball you measure its mass,  $m$ , and velocity,  $v$ , and estimate the uncertainties  $\delta m$  and  $\delta v$  associated with those quantities. The momentum is calculated using  $p = mv$ , but you need to also estimate the uncertainty on the momentum,  $\delta p$ , due to  $\delta m$  and  $\delta v$ . This section explains how to do this.

To keep everything straightforward, here is the list of variables and other nomenclature used throughout the section along with their descriptions:

- $x, y, z, u, w, \dots$  are variables whose values you have determined experimentally.
- $q = f(x, \dots, z)$  is a quantity which is calculated from  $x, y, z, \dots$  (e.g.  $q = x/y$ ).
- $\delta x$  is the *absolute* experimental uncertainty associated with  $x$  (i.e.  $x \pm \delta x$ ). It is always positive and it has the same units as  $x$ .
- $\delta x/|x|$  is the *relative* uncertainty associated with  $x$ . It has no units and it can be expressed as a percentage by multiplying it by 100.

Our goal is always to calculate the absolute uncertainty,  $\delta q$ , associated with  $q$ . However, sometimes it is necessary to first calculate the relative uncertainty,  $\delta q/|q|$ , and then calculate the absolute uncertainty with  $\delta q = |q| \times \delta q/|q|$ .

### 4.3.1 General formula to propagate uncertainties

This section describes where the rules in the following sections originate from. Skip on to Section 4.3.2 for the uncertainty propagation rules we expect you to use in this course.

The most general method (sometimes referred to as the differentiation method) for determining the uncertainty of a quantity  $q = f(x, \dots, z)$  is to calculate the partial derivatives of the function  $f$ , and to use the formula

$$\delta q = \sqrt{\left(\frac{\partial f}{\partial x} \delta x\right)^2 + \dots + \left(\frac{\partial f}{\partial z} \delta z\right)^2}. \quad (1)$$

This will work with any function but is only valid if all uncertainties are random and independent (i.e. not systematic). Another term for referring to this method is adding the uncertainties in quadrature (squaring each term in the sum and taking the square root of the sum).

Most of the calculations you will perform will involve only simple operations (addition, multiplication, square root, etc.); therefore, you will only have to remember how to deal with the specific examples listed below, where all formulae have been derived from Equation 1.

### 4.3.2 Addition and subtraction

If  $q$  consists of the sum and/or difference of  $x, y, \dots, z$  (i.e.  $q = x \pm \dots \pm z$ ), then the absolute uncertainty of  $q$  is given by:

$$\delta q = \sqrt{\delta x^2 + \dots + \delta z^2}. \quad (2)$$

For example, you take two steps forward (lengths  $L_1 = 56 \pm 1$  cm and  $L_2 = 47 \pm 2$  cm) and one step backward ( $L_3 = 52 \pm 1$  cm). In total, you have moved a distance  $L = L_1 + L_2 - L_3 = 51 \pm 3$  cm, since  $\delta L = \sqrt{\delta L_1^2 + \delta L_2^2 + \delta L_3^2} = 3$  cm. Remember to always round final uncertainties *up* (for more details on rounding uncertainties see Section 4.4).

### 4.3.3 Multiplication and division

If  $q$  consists of the products and/or quotients of  $x, y, u, v, \dots$  (i.e.  $q = x \times y \times \dots \div u \div v \div \dots$ ), then the relative uncertainty of  $q$  is given by:

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2 + \dots + \left(\frac{\delta u}{|u|}\right)^2 + \left(\frac{\delta v}{|v|}\right)^2 + \dots} \quad (3)$$

For example, suppose when you watch TV, your heart beats every  $\tau = 1.0 \pm 0.1$  s. Then during an episode of Buffy the Vampire Slayer, with duration  $T = 45 \pm 1$  min =  $2700 \pm 60$  s, your heart beats  $N = T/\tau = 2700 \pm 300$  times, since  $\delta N = |N| \times \sqrt{(\delta T/|T|)^2 + (\delta \tau/|\tau|)^2} \simeq 300$  (see Section 4.4 for more details on rounding to the correct number of significant figures).

#### 4.3.4 Multiplication or division by only a constant

If  $q$  is the product or quotient of a measured quantity and **only** an exactly known constant then the absolute uncertainty of  $q$  is the absolute uncertainty on the measured quantity multiplied or divided by the constant. For example,  $q = ax$  where you measure  $x$  and its uncertainty, and  $a$  is a constant known with infinite precision such that  $\delta a = 0$ . So for this example, the absolute uncertainty of  $q$  is given by

$$\delta q = a\delta x. \quad (4)$$

An example with numbers is that you measure the radius of a circle to be  $r = 45 \pm 1$  cm. Then the diameter of this circle is  $D = 2r = 90 \pm 2$  cm, since  $\delta D = 2\delta r = 2$  cm.

#### 4.3.5 Powers

If  $q$  consists of a measured quantity,  $x$ , with uncertainty raised to a power,  $n$ , that is known exactly (i.e.  $q = x^n$ ) then the relative uncertainty of  $q$  is given by

$$\frac{\delta q}{|q|} = |n| \frac{\delta x}{|x|}. \quad (5)$$

For example, you measure the length of one side of a square floor tile to be  $l = 10 \pm 3$  cm. Then the area of the tile is  $A = l^2 = 100 \pm 60$  cm<sup>2</sup> since  $\delta A = |2A| \times (\delta l/|l|) = 60$  cm<sup>2</sup>.

#### 4.3.6 Special functions

You may occasionally need to propagate uncertainties through special mathematical functions. Here are rules for some of the more common functions:

- Sine, like  $q = \sin \theta$ , will have absolute uncertainty given by  $\delta q = \cos \theta \delta \theta$ .
  - The same rule applies for cosine, just swap sin for cos and cos for sin.
  - Note that both  $\theta$  and  $\delta \theta$  must be in radians.
- Exponentials, like  $q = e^x$ , will have absolute uncertainty given by  $\delta q = e^x \delta x$ .
- Logarithms, like  $q = \ln cx$ , will have absolute uncertainty given by  $\delta q = \delta x/|x|$ .
  - Note that base-ten logarithms must first be converted to base- $e$  via  $q = \log_{10} x = 0.43 \ln x$ .

#### 4.3.7 Averages

If  $\bar{x}$  is the average of a set of measurements then the spread in that set (and thus the uncertainty on the average) is the sample standard deviation  $\sigma$ . This is calculated as

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} \quad (6)$$

where  $N$  is the number of measurements in the set and  $x_i$  is the  $i^{\text{th}}$  measurement in the set. For a set with about five or more values, you should instead use the standard error on the mean, given by

$$SE = \frac{\sigma}{\sqrt{N}}. \quad (7)$$

Notice that if more measurements are included in the average then the uncertainty will decrease. This is a powerful tool in experimentation and why we have you repeat measurements. An example with numbers is given in Section 4.5.4.

## 4.4 Proper significant figures on experimental values

When solving Avenue, textbook and exam questions, you are not usually given uncertainties. In those cases, the rule for significant figures is that your answer should only have as many significant figures as the number with the smallest number of significant figures used in the calculation. For LON-CAPA questions, aside from the questions specifically on significant figures (assignment 1), please enter your answers with three (3) significant figures. For LON-CAPA assignment 1, the uncertainties need to be rounded to the first significant figures nearest value.

During labs, you will estimate the uncertainties associated with your experimental results and so a different rule is used. This rule is broken into two steps.

1. Always keep only one significant figure on the values of uncertainties. When rounding the uncertainty to truncate it to only one significant figure, always round up (to have a more conservative uncertainty, like discussed previously).
2. The measured value associated with an uncertainty should have the same number of decimal places as its uncertainty. In other words, the last significant figure in the measured value should be in the same decimal place as its associated uncertainty.

These two steps will give you the correct number of significant figures on your experimentally measured values and their uncertainties. It is important to always include the uncertainty associated with a measured quantity when reporting your values. Finally, keep all significant figures during intermediate calculations, and only round results to the correct number of significant figures when reporting the final result. This will avoid round-off errors.

Here are some examples that illustrate how significant figures work with uncertainties.

- $L = 1.12 \pm 0.05$  cm is **correct** because the uncertainty has only one significant figure (in the hundredths place) and the measurement has the significant figures out to the same decimal place (hundredths).
- $L = 1.12354 \pm 0.05$  cm is **incorrect** because the measurement has too many significant figures compared to the last decimal place of the uncertainty.
- $L = 1.1 \pm 0.05$  cm is **incorrect** because the measurement has too few significant figures compared to the last decimal place of the uncertainty.

- $L = 1.12354 \pm 0.05546$  cm is **incorrect** because the uncertainty has more than one significant figure.
- $L = 1.124 \pm 0.050$  cm is **incorrect** because the uncertainty again has more than one significant figure.

## 4.5 Example uncertainty calculations with proper significant figures

### 4.5.1 Addition with multiple uncertainties and multiplication by a constant

Find the perimeter of a rectangle whose length,  $L$ , and width,  $W$ , are measured to be

$$L = 23.4 \pm 0.3 \text{ cm} \qquad W = 11.34 \pm 0.09 \text{ cm}$$

The perimeter,  $P$ , is

$$\begin{aligned} P &= 2(L + W) \\ &= 2(23.4 \text{ cm} + 11.34 \text{ cm}) \\ &= 69.48 \text{ cm}. \end{aligned}$$

The absolute uncertainty on the perimeter,  $\delta P$ , is

$$\begin{aligned} \delta P &= 2 * \sqrt{\delta L^2 + \delta W^2} \\ &= 2 * \sqrt{(0.3 \text{ cm})^2 + (0.09 \text{ cm})^2} \\ &= 0.626 \text{ cm} \\ &\approx 0.6 \text{ cm (rounded to one significant figure)}. \end{aligned}$$

So the final answer should be reported as  $P = 69.5 \pm 0.6$  cm, where both the measurement and uncertainty are quoted out to the tenth decimal place.

### 4.5.2 Multiplication and division with multiple uncertainties

You measure that it takes an object  $t = 8.3 \pm 0.5$  s to fall a distance of  $h = 30 \pm 3$  cm. To find the average speed,  $\bar{v}$ , we use the equation

$$\begin{aligned} \bar{v} &= \frac{h}{t} \\ &= \frac{30 \text{ cm}}{8.3 \text{ s}} \\ &\approx 3.61446 \text{ cm/s}. \end{aligned}$$

The relative uncertainty on the average speed,  $\delta\bar{v}/|\bar{v}|$ , is

$$\begin{aligned}\frac{\delta\bar{v}}{|\bar{v}|} &= \sqrt{\left(\frac{\delta h}{|h|}\right)^2 + \left(\frac{\delta t}{|t|}\right)^2} \\ &= \sqrt{\left(\frac{3 \text{ cm}}{30 \text{ cm}}\right)^2 + \left(\frac{0.5 \text{ s}}{8.3 \text{ s}}\right)^2} \\ &\approx 0.11674.\end{aligned}$$

Notice that there are no units on the result because they cancel out for relative uncertainty. What we really want, however, is the absolute uncertainty on the average speed so we do one more step to get

$$\begin{aligned}\delta\bar{v} &= |\bar{v}| \frac{\delta\bar{v}}{|\bar{v}|} \\ &= (3.61446 \text{ cm/s})(0.11674) \\ &\approx 0.42196 \text{ cm/s} \\ &\approx 0.5 \text{ cm/s}.\end{aligned}$$

The final result should be reported as  $\bar{v} = 3.6 \pm 0.5 \text{ cm/s}$ . We use the absolute uncertainty because we are reporting the average speed as a quantity that has an uncertainty associated to it.

### 4.5.3 Multiplication of measurements with uncertainty and a constant

You are sewing a skirt, and go to the store to buy fabric. The cost of fabric is \$9.98 per square meter. You order a piece that is  $w = 1.50 \pm 0.01 \text{ m}$  by  $l = 2.75 \pm 0.01 \text{ m}$ , and want to determine what the total cost will be, and the uncertainty on the cost. To calculate the total cost of fabric, we use the equation

$$\begin{aligned}\text{Cost} &= \text{rate} * \text{area} \\ &= \text{rate} * l * w \\ &= \$9.98 / \text{m}^2 (2.75 \text{ m})(1.50 \text{ m}) \\ &= \$9.98 / \text{m}^2 (4.125 \text{ m}^2) \\ &\approx \$41.168\end{aligned}$$

The relative uncertainty on the total fabric area,  $\delta A/|A|$ , is

$$\begin{aligned}\frac{\delta A}{|A|} &= \sqrt{\left(\frac{\delta l}{|l|}\right)^2 + \left(\frac{\delta w}{|w|}\right)^2} \\ &= \sqrt{\left(\frac{0.01 \text{ m}}{2.75 \text{ m}}\right)^2 + \left(\frac{0.01 \text{ m}}{1.5 \text{ m}}\right)^2} \\ &\approx 0.0075939\end{aligned}$$

and the absolute uncertainty,  $\delta A$ , is then

$$\begin{aligned}\delta A &= |A| \frac{\delta A}{|A|} \\ &= (4.125 \text{ m}^2)(0.0075939) \\ &\approx 0.0313248 \text{ m}^2.\end{aligned}$$

to find the absolute uncertainty on the cost  $\delta C$ , multiply the absolute uncertainty of the area  $\delta A$  by the constant rate:

$$\begin{aligned}\delta C &= \text{rate} * \delta A \\ &= \$9.98 / \text{m}^2 * 0.0313248 \text{ m}^2 \\ &\approx \$0.312622 \\ &\approx \$0.3\end{aligned}$$

The final cost should be reported as  $Cost = \$41.2 \pm 0.3$ . Note that the uncertainty should have only one significant figure and the result should be quoted out to the same decimal place.

#### 4.5.4 Average of five measurements

Five students measure the length of a book, each using their own ruler, and their results are

$$L_1 = 23.8 \text{ cm}, \quad L_2 = 24.0 \text{ cm}, \quad L_3 = 23.6 \text{ cm}, \quad L_4 = 23.9 \text{ cm}, \quad \text{and} \quad L_5 = 23.7 \text{ cm}.$$

The average of the measurements is

$$\begin{aligned}\bar{L} &= \frac{L_1 + L_2 + L_3 + L_4 + L_5}{5} \\ &= \frac{23.8 \text{ cm} + 24.0 \text{ cm} + 23.6 \text{ cm} + 23.9 \text{ cm} + 23.7 \text{ cm}}{5} \\ &= 23.8 \text{ cm}.\end{aligned}$$

We will use the standard error on the mean to estimate the uncertainty on the average. Since five different people using different measurement tools recorded the values, the variations should be independent and random. The standard error on the mean is

$$SE = \frac{\sigma}{\sqrt{N}}$$

where  $\sigma$  is the sample standard deviation and  $N$  is the number of trials. We first calculate the sample standard deviation as

$$\begin{aligned}\sigma &= \sqrt{\frac{(L_1 - \bar{L})^2 + (L_2 - \bar{L})^2 + (L_3 - \bar{L})^2 + (L_4 - \bar{L})^2 + (L_5 - \bar{L})^2}{N - 1}} \\ &= \left[ \frac{1}{5 - 1} \left( (23.8 \text{ cm} - 23.8 \text{ cm})^2 + (24.0 \text{ cm} - 23.8 \text{ cm})^2 + (23.6 \text{ cm} - 23.8 \text{ cm})^2 \right. \right. \\ &\quad \left. \left. + (23.9 \text{ cm} - 23.8 \text{ cm})^2 + (23.7 \text{ cm} - 23.8 \text{ cm})^2 \right) \right]^{1/2} \\ &\approx 0.15811 \text{ cm}.\end{aligned}$$

Now the standard error on the mean (and thus the absolute uncertainty on the average) is

$$\begin{aligned}SE &= \frac{\sigma}{\sqrt{N}} \\ &= \frac{0.15811 \text{ cm}}{\sqrt{5}} \\ &\approx 0.07071 \text{ cm} \\ &\approx 0.08 \text{ cm}.\end{aligned}$$

The final answer we report is  $\bar{L} = 23.80 \pm 0.08 \text{ cm}$ . We could also use the sample standard deviation as our answer, but we chose to use the standard error since we had five measurements. When using the standard deviation or standard error as your uncertainty for a set of data, you should always report it with the average of that data.

## 4.6 Analyzing graphs

### 4.6.1 What makes a clear graph?

It is common to present scientific results graphically. Discrepancies can clearly appear with a quick glance because the human mind is very well adapted to visual pattern recognition. The first step in analyzing your graphs is to first make them, and to make sure all of the necessary components are included to make them as clear as possible. Make sure all graphs contain these items:

- Correct variables on each axis in appropriate units.
- Error bars on both x and y axes (when asked for).
- A presented equation for the line of best-fit when asked to do linear regression.
- Careful spacing so no pertinent data or labels are blocked.
- Axis ranges that minimize blank space (see Figure 1 below).



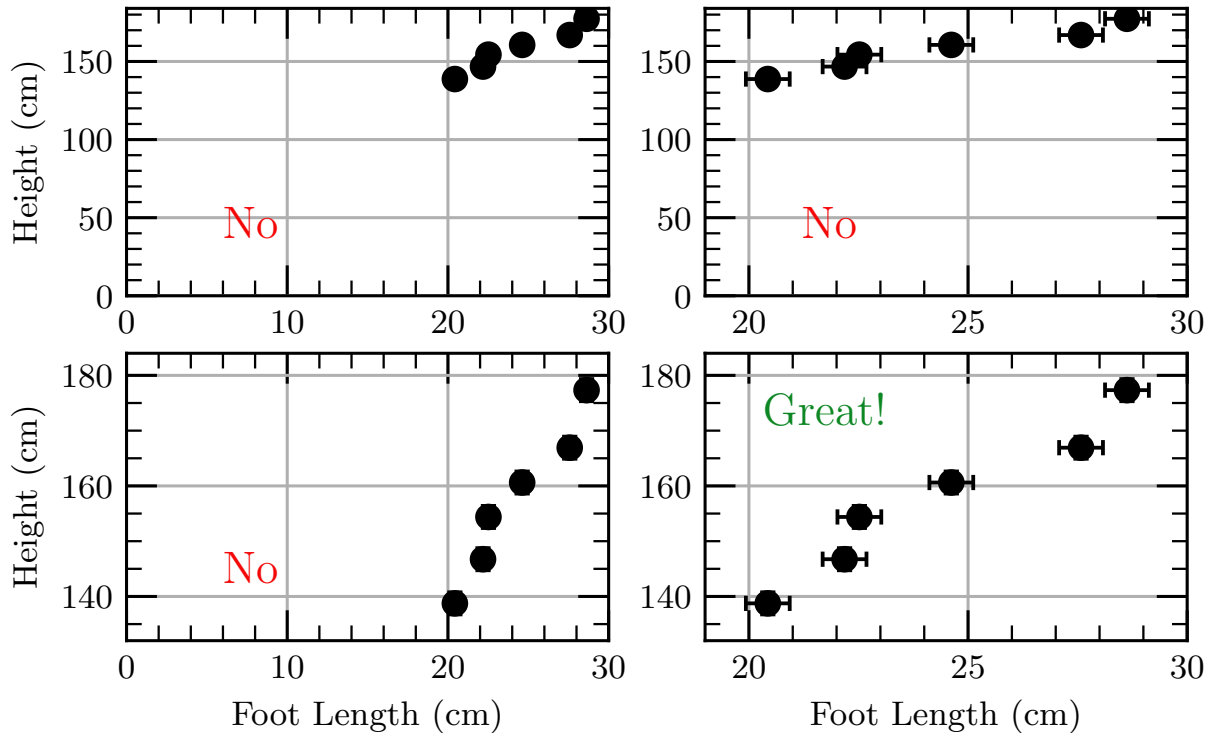


Figure 1: A demonstration of what is meant by minimizing blank space in figures. Note that each panel shows the same data, but the bottom right panel is the easiest to interpret, and looks the best. When creating plots, make sure the axes ranges have been set to include all the data, like in the bottom right panel. Also, remember to leave *a little* space around the edges, and to not put the data *on* the axes.

It is common for plotting software to by default set the axes ranges so that the bottom left corner is at  $(x, y) = (0, 0)$ . However, in most situations, like in our examples from this section, the data are not particularly clustered at  $(0, 0)$ , so including  $(0, 0)$  introduces a lot of blank space that makes it harder to read the data. Figure 1 visually demonstrates some examples of poor practices and best practices when scaling axes in figures. Figure 2 shows another example of what is expected for graphs, made from the data in Table 1.

#### 4.6.2 Logarithmic data

In some of the experiments you will be asked to analyze log-log graphs (i.e. graphs where the logarithms of the data are plotted) to help reveal power-law relationships between measurements. In the case of the simple pendulum the period  $T$  is proportional to the square root of the length  $L$  of the string,

$$T = 2\pi \left( \frac{L}{g} \right)^{\frac{1}{2}}$$

This is an example of a power law relation, with a power (exponent) of  $\frac{1}{2}$ . To test this

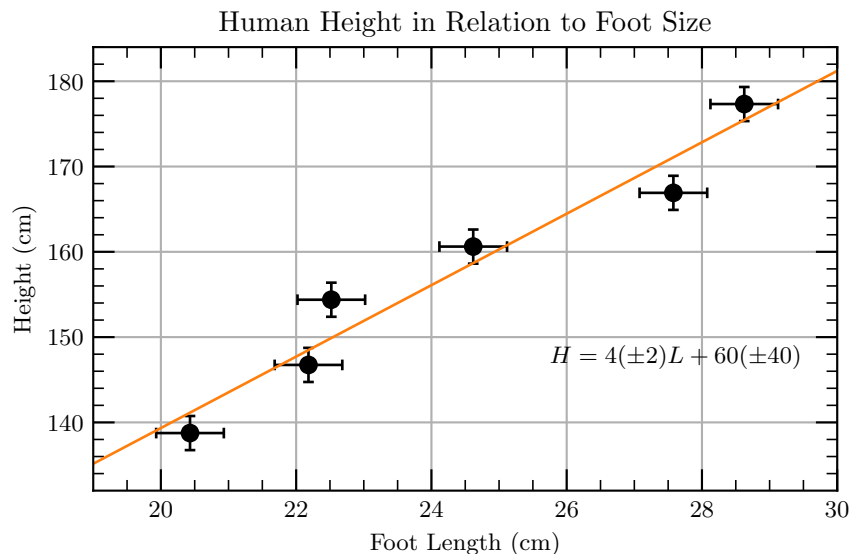


Figure 2: An example of a proper scientific graph featuring a title, axis labels, data points (black circles), error bars (black lines attached to data points), trend line (orange line passing through the data points), and trend line equation with uncertainties. Note: we will not need to report the uncertainties associated with trend lines in these courses.

Table 1: Example measurements of human foot lengths and heights. While all uncertainties on foot length are 0.5 cm and on height are 2 cm, all uncertainties on a quantity do not have to be the same, in general. Graphs to visualize and analyze this data are shown in Figure 2.

Foot Length (cm)	Foot Length Uncertainty (cm)	Height (cm)	Height Uncertainty (cm)
20.4	0.5	139	2
22.2	0.5	147	2
22.5	0.5	154	2
24.6	0.5	161	2
27.6	0.5	167	2
28.6	0.5	177	2

relation using a straight line,  $T$  must be plotted against  $\sqrt{L}$  or  $T^2$  against  $L$ .

However, if the graph is not in agreement with the particular power law you expected, this type of plot would not indicate the correct exponent. Another way to test power law relations is to use a log-log plot. As an example, suppose you wish to determine whether there is a power law relation between the set of measured quantities  $x$  and  $y$ , i.e., to test whether the relation

$$y = ax^n$$

holds for some unknown constants  $a$  and  $n$ . The power  $n$  can be made more visible by taking logarithms of both sides (it does not matter what logarithm base is used):

$$\begin{aligned} \log(y) &= \log(ax^n) = \log(a) + \log(x^n) \\ &= \log(a) + n\log(x) \end{aligned} \tag{8}$$

Hence, if there is a power law relation between  $x$  and  $y$ , a plot of  $\log(y)$  against  $\log(x)$  could be fit by a straight line of slope  $n$  and intercept  $\log(a)$ . Of course, uncertainty ranges are needed to make such a decision. The methods for estimating the uncertainties in logs in terms of the uncertainties in their arguments have been described above.

For example, for the simple pendulum the theory predicts that a plot of  $\log(T)$  against  $\log(L)$  should be fit by a straight line of slope 0.5, as taking logs to any base of the theoretical prediction given above yields

$$\log T = \log\left(\frac{2}{\sqrt{g}}\right) + \frac{1}{2}\log L.$$

Note that although a logarithm is a pure number with no units, it is important to indicate on a graph the units of its argument. For instance, if natural logarithms are used, the label on the  $\ln(L)$  axis for the above example of the simple pendulum could read:

“ $\ln(L)$  with  $L$  in mm”

Finally, it should be emphasized that the log-log plot of the data for  $x$  and  $y$  may turn out to have a curved shape which would be poorly fitted by a straight line. In that case you would conclude that the simple power law relation between  $x$  and  $y$  does not appear to be valid. As always, you must consider the experimental uncertainties, using error bars on the graph, when drawing conclusions.

## 4.7 Drawing Conclusions

In these laboratory experiments, you will often be asked whether a theoretical and experimental result are consistent. Suppose you have the results  $Q_{theory}$  and  $Q_{expt} \pm \delta Q$  for a given quantity. If the value for  $Q_{theory}$  lies in the range of  $Q_{expt} \pm \delta Q$ , you say that “ $Q_{theory}$  and  $Q_{expt}$  agree within experimental uncertainty”.

What happens if they don’t agree (and you’ve approximated the effect of random errors properly)? Either the theory is invalid, or there are systematic errors in the equipment or experimental design that prevented you from obtaining an accurate result. If this is the case, you should make note of where in the experiment you feel that systematic error exists, how large it might reasonably be, and how you would correct it.

Note: If you have two experimental results that you are comparing, each with its own uncertainty, they need only agree somewhere within the range of their uncertainties for you to conclude that they are consistent with each other.

Many times in the lab, we will be making conclusions from various graphs generated throughout the experiment. This will often involve finding the line of best fit for a set of data and determining the constants of a power law relationship. This is also an acceptable method for drawing a positive conclusion from the experiment.

## 5 Microsoft Excel and Simple Data Analysis

Since the advent of computers in the workplace, automation has taken over many tasks that would have been deemed tedious in past decades. As engineers in the workforce, you will be expected to keep up with modern changes in technology and how they can be applied to solving problems around the world. Part of the goals in the Physics 1D03/1E03 labs will be to introduce you to spreadsheet software, and how they can be used to simplify many tasks.

Spreadsheet software is software that is designed to record, organize, and analyze data of all types within a tabular format. They were originally developed as digital analogs to written paper worksheets but have since become fundamental to the workforce and have all but replaced their physical counterparts. This type of software is now so ubiquitous in many different facets of life, it is often a default choice for simple data analysis or organization.

There are many different types of spreadsheet software available on the market. As examples you may have encountered already, Google has Google Sheets, Apple has Numbers, Linux's base spreadsheet is LibreOffice, and Microsoft has Excel. Many of these software's have varying benefits and detriments to them. Google Sheets is great for sharing amongst colleagues since it was designed to be integrated within a cloud-based storage system, while Excel provides some of the best data organization and built-in functionality.

Microsoft Excel will be the primary spreadsheet software used within these labs and we will only be working with numerical data. Although many of these software have different commands for different functions, or have different limitations, the lessons learned will be easily transferable to any reputable spreadsheet software. Keep in mind that all of the labs will be taught with Excel and if you wish to do any of these things with a different spreadsheet software, the commands or functionality may be different.

### 5.1 Excel's User Interface

For those familiar with word processing programs such as Microsoft Word, the ribbon tabs and bars will look very similar in Excel. The ribbon is a row of tabs and icons at the top of the Excel window which allows you to quickly find, understand, and use commands for completing certain tasks. As shown in Figure 3, the ribbon is broken down into a series of tabs with subcategories of bars. Excel provides many interesting and unique options for users to customize their work! You can even customize the ribbon to your own settings. For these labs, we will be using the default ribbon.

The workbook and the worksheets within the workbook are the most important part of Excel and it is where the primary applications of this software will come from. These worksheets are comprised of a series of columns labeled with the alphabet and rows labeled with numbers for each cell. Each cell within the work sheet can be quickly and easily referenced by their associated column and row designation. The workbook is simply the name for the set of worksheets within an Excel file. An example of this is shown in Figure 4 where the workbook is comprised of three worksheets.

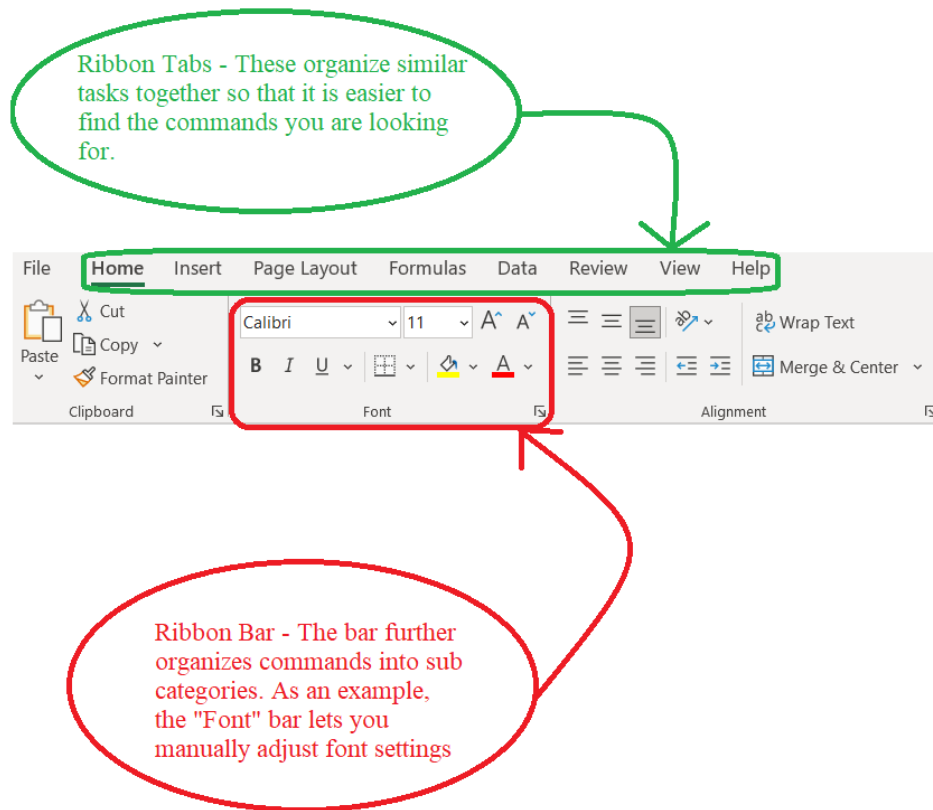


Figure 3: Image showing the ribbon in Excel which provides shortcuts to commands.

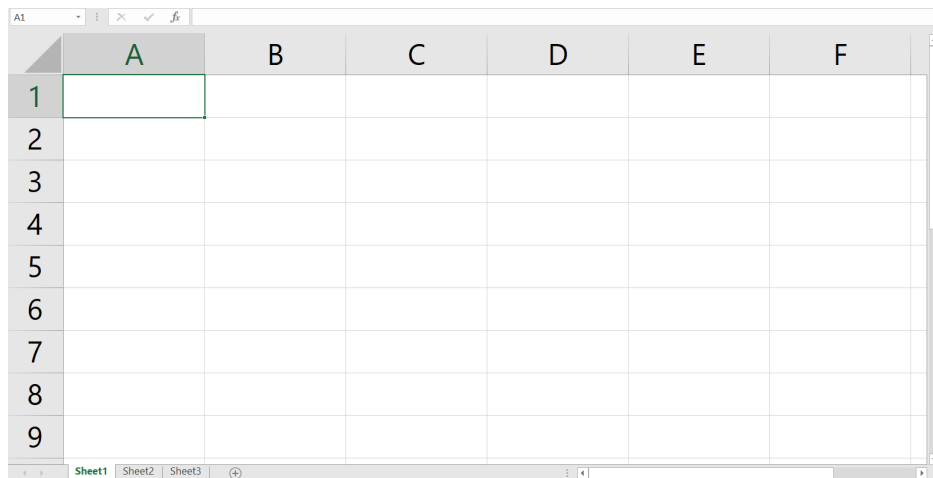


Figure 4: Zoomed in image of the default workbook within Microsoft Excel. Notice how each cell is labeled by a column with a letter, and a row with a number in that order. This is indicated in the top right part of the image where the cell is highlighted and labeled as “A1”. This allows for one to reference the cell quickly and easily within the worksheet.

## 5.2 Data Organization and Operations

Whenever you are presenting data, one of the key goals is to make that data easily reproducible and read by others. Excel's user interface provides quite a bit of support for this with intuitive labeling functionality. For our purposes, we will go through how to present numerical data and how to use this in-built functionality for various tasks such as arithmetic operations on cells and generating plots.

A standard way to reference the data you are working with is to label the columns for that data in bold. This indicates to the reader what that column represents, and its given units or uncertainty.

To best demonstrate this labeling convention and the basic arithmetic operations supported between cells, consider the spreadsheet given in Figure 5. Each cell is referenced by its corresponding column label and row label in that order. So instead of having to manually enter each value every time an arithmetic operation is done (like what is done by hand), you can just type that cell's reference label and apply operations between them. The four basic arithmetic operations are shown in Figure 5 with their corresponding operation in quotations. Note: For numerical values, Excel supports all legal operations (no division by zero or square root of negative numbers) for real numbers.

Equation Editor - The equation editor allows you to apply any mathematical operation on numerical values between cells and it will appear in the cell of choice (in this case D2).

	A	B	C	D	E	F	G
1	Trails	Distance 1 (m)	Distance 2 (m)	Addition "+"	Subtraction "-"	Multiplication "*"	Division "/"
2	1	4	42	=B2+C2	-38	168	0.095238095
3	2	-12.32	6.25	-6.07	-18.57	-77	-1.9712
4	3	37	5.142311515	42.14231152	31.85768848	190.2655261	7.195207815
5	4	3.141592654	53	56.14159265	-49.85840735	166.5044107	0.059275333

Figure 5: Excel spreadsheet showing the basic labeling conventions and arithmetic operations between cells. The two quantities being examined here “Distance 1” and “Distance 2” both have an associated unit of meters which is indicated by (m). The arithmetic operations are being demonstrated through columns D-G along with the corresponding command for them in quotations. Notice how operations can be applied within cells simply by referencing the cells address. Note: The significant figures in these columns are explicitly ignored to show how Excel supports arithmetic operations of all real numbers.

Excel supports many cool and interesting functions as well. Here is a non exhaustive list of some important ones.

- Trigonometric functions:
  - Tangent = TAN()
  - Arc Sine = ASIN()
  - Secant = SEC()

- Logarithmic functions: Natural logarithm = LN()
- Square root = SQRT()
- Sums between values i.e.  $A1+A2+\dots+A_n=\text{SUM}(A1:A_n)$
- The average for a range of values as  $(A1+A2+\dots+A_n)/n=\text{AVG}(A1:A_n)$

I encourage you to look into this yourself and see the many types of functions within Excel!

## 5.3 Automating Data Entry and Operations

### 5.3.1 Data Entry

Many of the tasks discussed so far can be done rather easily by hand. But what if we wanted to apply these operations to thousands or millions of values? This task would quickly get out of hand and be quite tedious. Thankfully, Excel provides great ways to automate data entry and apply operations between multiple cells.

As an example, suppose we wanted to calculate the period  $T$  of a simple pendulum from 200 values of the length of a string  $L$ , equally spaced between 0.5 meters and 2 meters through the formula

$$T = 2\pi\left(\frac{L}{g}\right)^{\frac{1}{2}}.$$

Calculating the value of  $T$  for 200 different values of  $L$  would be extremely tedious to do by hand and would likely lead to making a mistake. Let's see how we can make this task simpler through Excel.

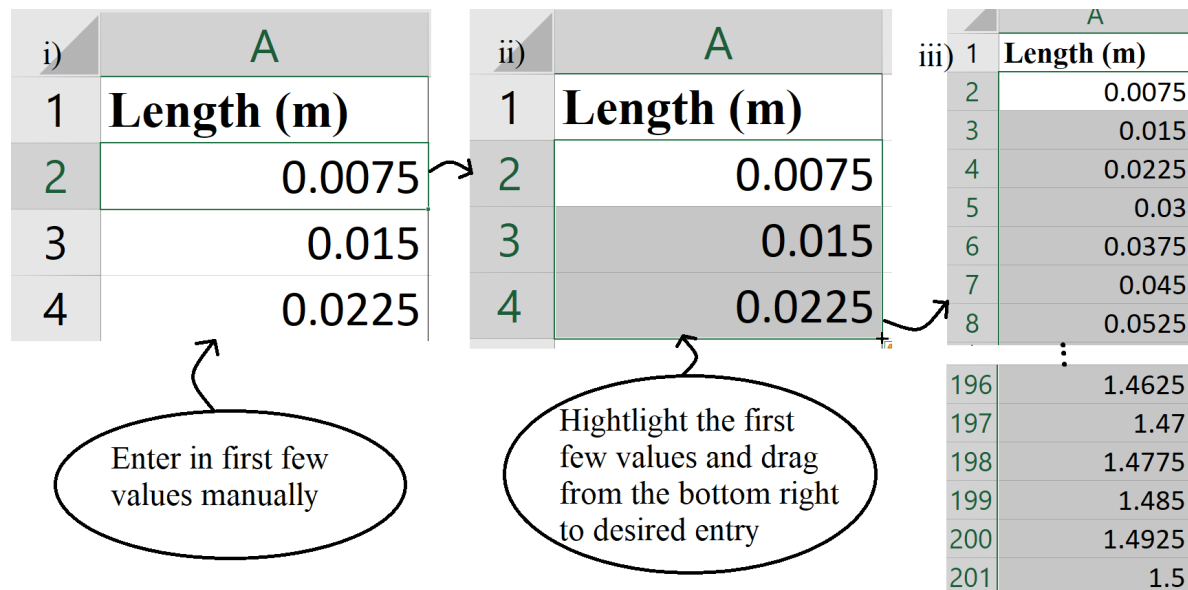


Figure 6: Demonstration on automating data entry with a known pattern. The first few values (0.0075, 0.015, and 0.0225) are calculated by hand, and then the rest are automatically filled by Excel to the desired cell (in this case A201).

The process for automating this task is illustrated in Figure 6 and listed below:

- (i) Enter the first few values in manually.
  - (a) Since each value of L is equally spaced between 0.5 and 2 meters, this means each length is  $(2-0.5)\text{m}/200=0.0075\text{m}=7.5\text{mm}$  away from the next value.
- (ii) Highlight the selected area of interest.
- (iii) Drag the highlighted area to the desired row. Excel will calculate each cell after the highlighted area based on a recursive pattern from the previous cells.
  - (a) In this case, since the difference between each cell is 7.5mm, Excel will calculate A5 to be  $A5=A4+7.5\text{mm}=0.03\text{m}$ . A6 will be  $A6=A5+7.5\text{mm}=0.0375\text{m}$ . This pattern will continue to the desired entry which in this case is A201.

As such, we now have a column of equidistant lengths with 200 elements!

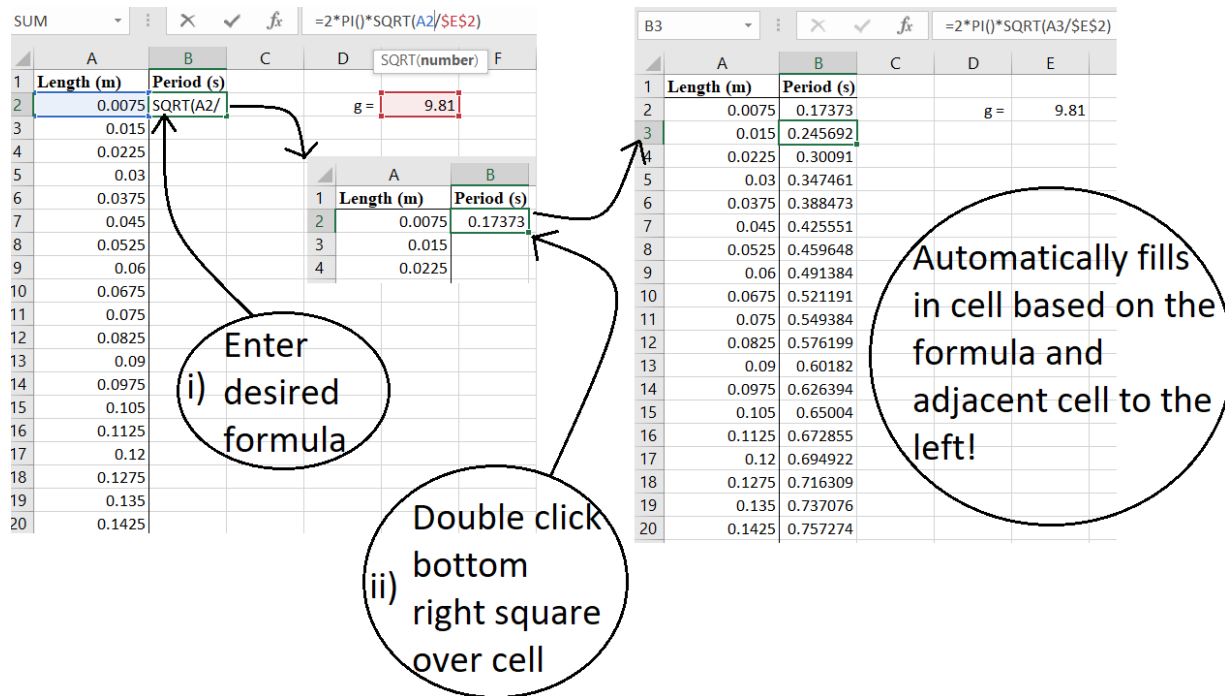


Figure 7: Demonstration on automating data entry with a known pattern. The first few values (0.0075, 0.015, and 0.0225) are calculated by hand, and then the rest are automatically filled by Excel to the desired cell (in this case A201).



### 5.3.2 Data Operations

Now if we want to calculate the period for each length, we can use the formula given above on each length with a constant value of  $g = 9.81\text{m/s}^2$ . All we need to do is create a second column and enter the desired formula for each value of length. This is easy to do in Excel and the process is illustrated in Figure 7 and listed below.

- (i) Enter the desired formula into the cell of interest. Here we want the value of period in B2 to be associated with the length for A2.
- (ii) Double click the bottom right square over the highlighted cell which is B2 in this case. From here, Excel will automatically calculate this formula from column A into column B.
  - (a) Note: Even though the corresponding row value for B changes based on A (i.e. B3 is calculated from A3, B4 is calculated from A4, ect...), they are all calculated with the constant value  $g = 9.81\text{ m/s}^2$  by using the command  $\$E\$2$ . This command tells Excel to keep the value the same for all values in the column.

Now we have a calculated value for period from each length!

## 5.4 Graphing Data in Excel

As discussed earlier, graphing data is often a great way to recognize patterns within a set of data. In these experiments, you will be using Excel to graph the data you collected within the labs. At the start, these graphs will be automatically generated for you, but later on, you will be expected to make your own graphs.

Within Excel, there are multiple ways to plot data from a worksheet. I will demonstrate the most secure way to generate a plot by using the data set we generated for the period of a simple pendulum. Given this data has 200 data points, it would be extremely difficult to plot them by hand as making a reasonable scale for 200 data points on a standard piece of graphing paper is almost impossible. Instead, we will let Excel do the work for us!

There are many different types of graphs that can be used in Excel (pie charts, bar graphs, scatter plots, ect...) but for the majority of this course, we will be using scatter plots. The procedure for creating a standard scatter plot in the most general way is outlined in Figure 8 and shown below:

- (i) Select the “Insert” tab and then the “Scatter” option. This will create a blank chart area which you can edit.
  - (a) Note: there are many different types of scatter plots you can generate. For the purposes of our labs, we will only be using the standard scatter plot without lines connecting them.
- (ii) Right click on the blank chart area and choose the “Select Data” option.

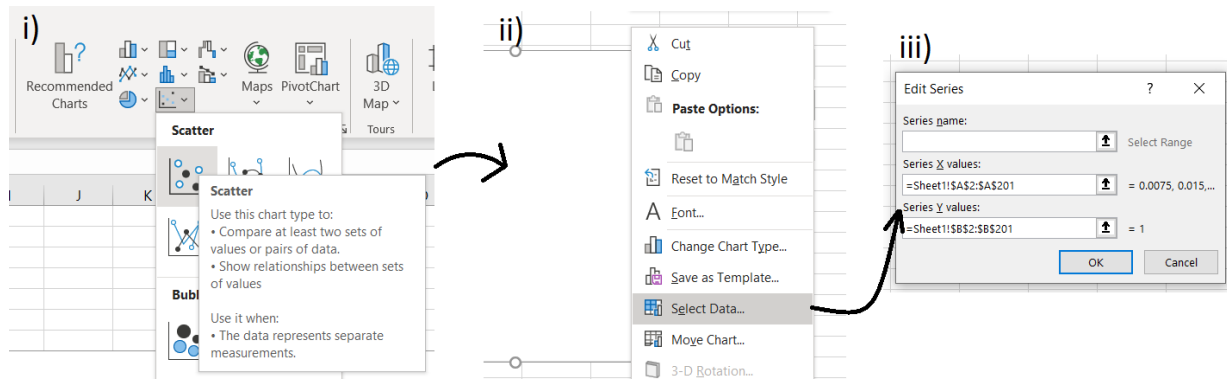


Figure 8: Demonstration on how to create a scatter plot from any set of data within Excel. For two columns of data, you can also generate a plot by highlighting both columns and inserting the desired plot while both columns are highlighted.

- (iii) From here, you can add a data series of your choosing by selecting the “Add” option within the “Select Data Source” window. This will allow you to explicitly select the data of your choice as the x and y axes.
- (a) An easy way to do this is to shift click the entire column under the “Series X values:” or “Series Y values:”.
  - (b) Another way to do this is to manually type in the desired data.
    - Note: this can be useful when you have multiple columns and only want to include some of the data.

This is the most secure method for creating a plot of any type from any set of data within Excel. An easier way to do this with two columns is to highlight both of the columns and insert the desired plot.

#### 5.4.1 Graph Titles and Labels in Excel

Every graph or plot in these labs will require proper titles and labels. At the start of the Physics 1D03 labs, the lab manual will explicitly describe how the labels and titles can be added to these graphs. The procedure for adding these graph features is fairly straight forward and described below.

1. Select the Chart area of interest.
2. Under the “Chart Design” tab, select the “Add Chart Element” and from there, choose which plot features you would like to add.

#### 5.4.2 Importing and Exporting data

Some of the labs will require you to import data from different sources into Excel. Luckily, Excel is made to handle importing and exporting data quite well. Any data file which is

in a comma separated value (csv) file or tab separated value (tsv) file can be imported into Excel with relative ease.

The simplest way to import a csv or tsv file is to open that file with Excel. You can however, manually import many types of data or queries through Excel. This can all be found under the data tab. For the purposes of this course, many of the input/output (I/O) commands will be described in the lab manual and will solely involve csv files.

## 5.5 Linear Regression (Line of Best Fit)

You may have heard the term linear regression in the past. In simple terms, this is the statistical name for finding the line of best fit from a set of data. This topic is quite in depth and many of its applications go beyond the scope of this course. We will however be finding the line of best fit for sets of data which has one dependent and independent variable with constants in order to verify power law relations.

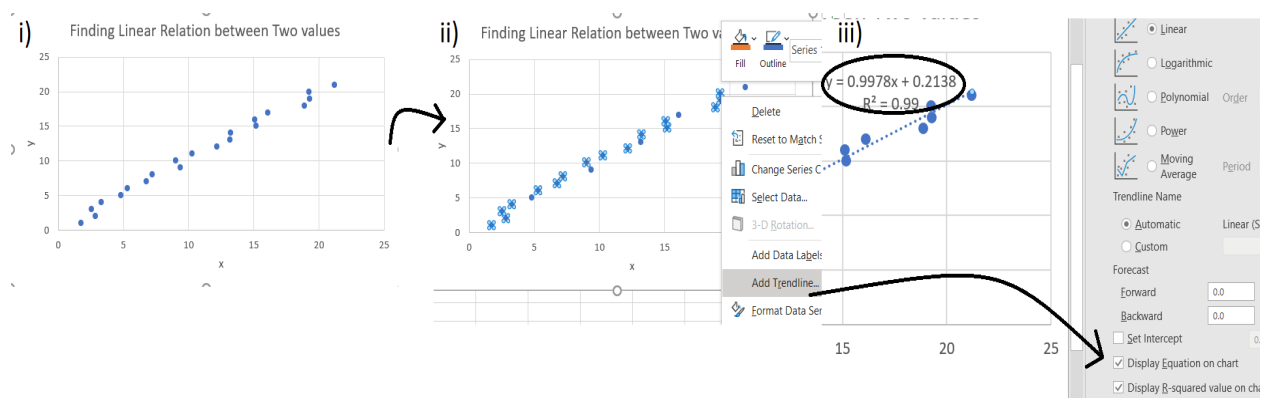


Figure 9: Demonstration on how to generate a line of best fit for two columns of data.

The data that we apply linear regression to will often be logarithmic in nature since the physical relations we study are often non-linear (to the power of something other than 1). Within Excel, we can easily find the line of best fit for two columns of data and plot the equation of the line. Figure 9 visualizes the process to insert a line of best fit for a set of data. The procedure is described as follows:

- (i) Right click on a data point within the chart area.
- (ii) Select the “Add Trendline...” option. This will display a line of best fit on the graph of interest.
- (iii) Select “Display Equation on chat” and “Display R-squared value on chart”.
  - (a) This will display the equation of the best fit line onto your chart along with the  $R^2$  value.
  - (b) Note: the  $R^2$  value is known as the coefficient of determination and it represents how well the linear line “fits” the data you are applying linear regression to. If the  $R^2$  is close to 1, it means it is a very good fit, if it is close to 0, it means it is a very poor fit.

Throughout these labs we will be finding the equation of the line for various physical variables in order to justify experimental results with theoretical predictions. The error associated with the line of best fit will not be discussed in these labs as the statistics go beyond the scope of this course. If you are interested in learning more about how to do this with Excel, I recommend looking further into the “LINEST” function.