CHAPTER 2. UNDERSTANDING MEASUREMENT

Enhanced Introductory College Chemistry

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Chapter Contents

Please visit the web version of Enhanced Introductory College Chemistry (https://ecampusontario.pressbooks.pub/enhancedchemistry/) to access the complete book, interactive activities and ancillary resources.

In this chapter, you will learn about

- Exact and uncertain measurements
- Numerical prefixes and equalities
- Analyzing numerical problems

To better support your learning, you should be familiar with the following concepts before starting this chapter:

- The metric system
- Basics of arithmetic

Measurements provide the macroscopic information that is the basis of most of the hypotheses, theories, and laws that describe the behaviour of matter and energy in both the macroscopic and microscopic domains of chemistry. Every measurement provides three kinds of information: the size or magnitude of the measurement (a number); a standard of comparison for the measurement (a unit); and an indication of the uncertainty of the measurement. While the number and unit are explicitly represented when a quantity is written, the uncertainty is an aspect of the measurement result that is more implicitly represented and will be discussed later.

Scientists in Action: Twelve Women in Chemistry

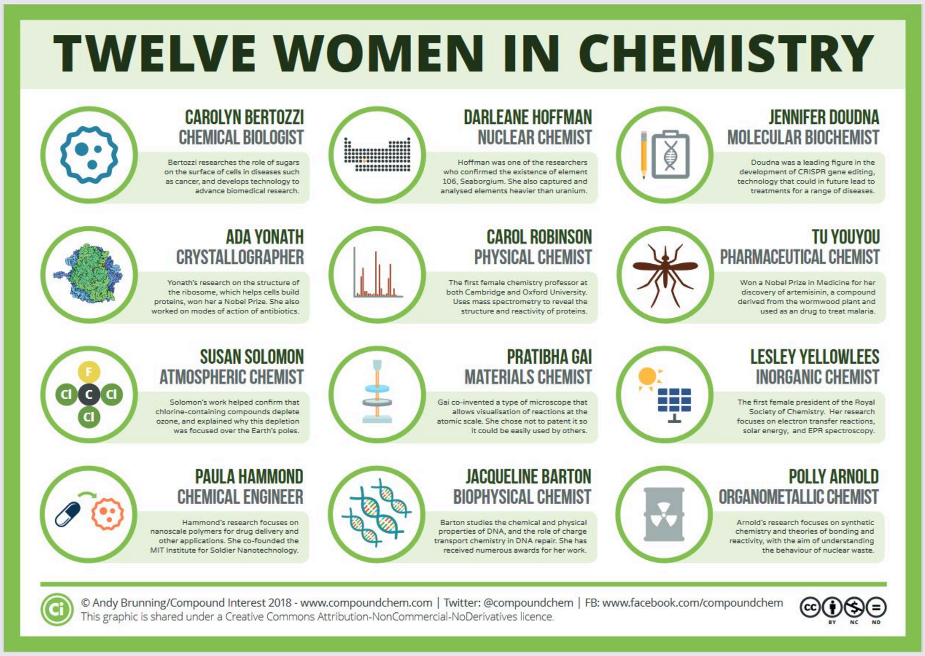

Figure 2a "Twelve Women in Chemistry" by Andy Brunning/Compound Interest is licensed under CC BY-NC-ND 4.0.

Attributions & References

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2.1 MEASUREMENTS

Learning Objectives

By the end of this section, you will be able to:

- Understand and apply fundamental measurements in scientific notation.
- Employ Systeme Internationale (SI) measurements

Scientific Notation

The number in the measurement can be represented in different ways, including decimal form and scientific notation. For example, the maximum takeoff weight of a Boeing 777-200ER airliner is 298,000 kilograms, which can also be written as 2.98×10^5 kg. The mass of the average mosquito is about 0.0000025 kilograms, which can be written as 2.5×10^{-6} kg.

Scientific Notation is a method to simplify very large and very small numbers by utilizing a base 10 exponential methodology. The in above examples given 298 000 kg is equivalent to 2.98 × 10 × 10 × 10 × 10 × 10 × 10 or $2.98{\times}10^5$ whereas, 0.0000025 kilograms, is equivalent to 2.5 × 1/10 × 1/10 × 1/10 × 1/10 × 1/10 × 1/10 or 2.5 x 10^{-6} kg. Using a scientific calculator effectively can reduce the introduction of calculation errors. For example, 298 000 can be inputted as 2.98 [EXP] 5 as scientific notation of the former example.

Units

Units, such as litres, pounds, and centimetres, are standards of comparison for measurements. When we buy a 2-litre bottle of a soft drink, we expect that the **volume** of the drink was measured, so it is two times larger than the volume that everyone agrees to be 1 litre. The meat used to prepare a 0.25-pound hamburger is measured so it weighs one-fourth as much as 1 pound. Without units, a number can be meaningless, confusing, or possibly life threatening. Suppose a doctor prescribes phenobarbital to control a patient's seizures and states a dosage of "100" without specifying units. Not only will this be confusing to the medical professional giving the dose, but the consequences can be dire: 100 mg given three times per day can be effective as an anticonvulsant, but a single dose of 100 g is more than 10 times the lethal amount.

We usually report the results of scientific measurements in **SI units**, an updated version of the metric system, using the units listed in Table 2.1a. Other units can be derived from these base units. The standards for these units are fixed by international agreement, and they are called the International System of Units or SI Units (from the French, *Le Système International d'Unités*). SI units have been used by the United States National Institute of Standards and Technology (NIST) since 1964.

Table 2.1a Base Units of the SI System

Property Measured	Name of Unit	Symbol of Unit
length	meter	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
electric current	ampere	A
amount of substance	mole	mol
luminous intensity	candela	cd

Sometimes we use units that are fractions or multiples of a base unit. Ice cream is sold in quarts (a familiar, non-SI base unit), pints (0.5 quart), or gallons (4 quarts). We also use fractions or multiples of units in the SI system, but these fractions or multiples are always powers of 10. Fractional or multiple SI units are named using a prefix and the name of the base unit. For example, a length of 1000 meters is also called a kilometre because the prefix *kilo* means "one thousand," which in scientific notation is 10^3 (1 kilometre = 1000 m = 10^3 m). The prefixes used and the powers to which 10 are raised are listed in Table 2.1b.

Table 2.1b Common Unit Prefixes

Prefix	Symbol	Factor	Example
femto	f	10^{-15}	1 femtosec ond (fs) = 1 × 10^{-15} s (0.000000000000001 s)
pico	p	10^{-12}	1 picometer (pm) = 1 × 10^{-12} m (0.000000000001 m)
nano	n	10^{-9}	4 nanograms (ng) = 4 × 10^{-9} g (0.000000004 g)
micro	µ	10^{-6}	1 microliter (µL) = 1 × 10^{-6} L (0.000001 L)
milli	m	10^{-3}	2 millimoles (mmol) = 2 × 10^{-3} mol (0.002 mol)
centi	c	10^{-2}	7 centimeters (cm) = 7 × 10^{-2} m (0.07 m)
deci	d	10^{-1}	1 deciliter (dL) = 1 × 10^{-1} L (0.1 L)
kilo	k	10^{3}	1 kilometer (km) = 1 × 10^{3} m (1000 m)
mega	M	10^{6}	3 megahertz (MHz) = 3 × 10^{6} Hz (3,000,000 Hz)
giga	G	10^{9}	8 gigayears (Gyr) = 8 × 10^{9} yr (8,000,000,000 Gyr)
tera	T	10^{12}	5 terawatts (TW) = 5 × 10^{12} W (5,000,000,000,000 W)

SI Base Units

The initial units of the metric system, which eventually evolved into the SI system, were established in France during the French Revolution. The original standards for the meter and the kilogram were adopted there in 1799 and eventually by other countries. This section introduces four of the SI base units commonly used in chemistry. Other SI units will be introduced in subsequent chapters.

Length

The standard unit of **length** in both the SI and original metric systems is the **meter (m)**. A meter was originally specified as 1/10,000,000 of the distance from the North Pole to the equator. It is now defined as the distance light in a vacuum travels in 1/299,792,458 of a second. A meter is about 3 inches longer than a yard (Figure 2.1a); one meter is about 39.37 inches or 1.094 yards. Longer distances are often reported in kilometres (1 km = 1000 m = 10^{3} m), whereas shorter distances can be reported in centimetres (1 cm = 0.01 m = 10^{-2} m) or millimetres (1 mm = 0.001 m = 10^{-3} m).

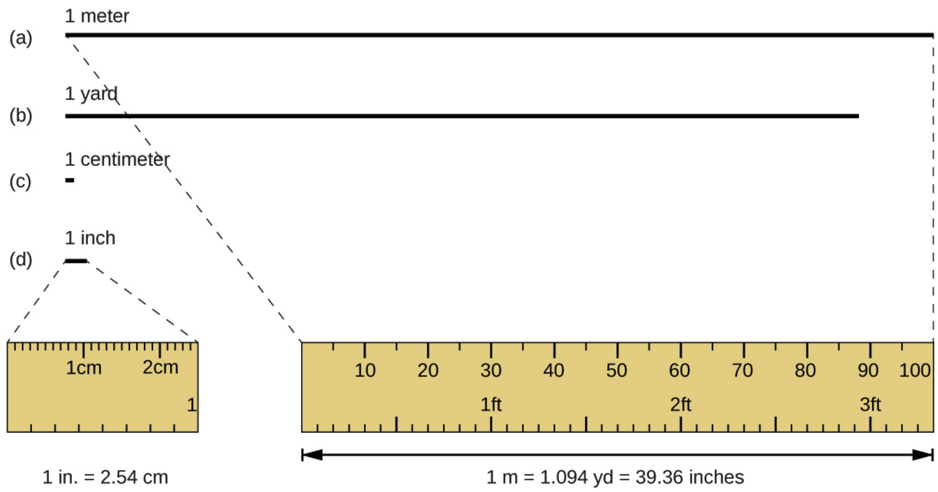

Figure 2.1a The relative lengths of 1 m, 1 yd, 1 cm, and 1 in. are shown (not actual size), as well as comparisons of 2.54 cm and 1 in., and of 1 m and 1.094 yd (credit: *Chemistry (OpenStax)*, CC BY 4.0).

Mass

The standard unit of mass in the SI system is the **kilogram (kg)**. A kilogram was originally defined as the mass of a litre of water (a cube of water with an edge length of exactly 0.1 meter). It is now defined by a certain cylinder of platinum-iridium alloy, which is kept in France (Figure 2.1b). Any object with the same mass as this cylinder is said to have a mass of 1 kilogram. One kilogram is about 2.2 pounds. The gram (g) is exactly equal to 1/1000 of the mass of the kilogram (10^{-3} kg).

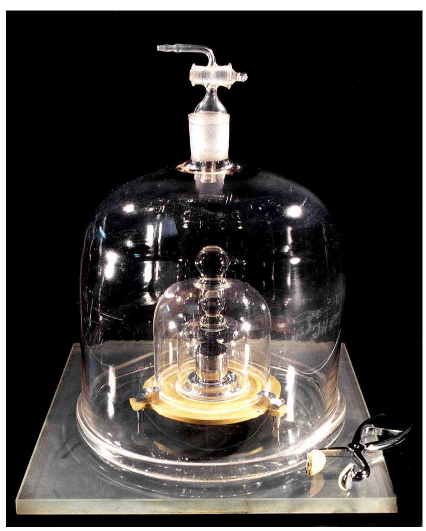

Figure 2.1b This replica prototype kilogram is housed at the National Institute of Standards and Technology (NIST) in Maryland. (credit: photo by BIPM, CC BY-SA 3.0)

Temperature

Temperature is an intensive property. The SI unit of temperature is the **kelvin (K)**. The IUPAC convention is to use kelvin (all lowercase) for the word, K (uppercase) for the unit symbol, and neither the word "degree" nor the degree symbol (°). The degree **Celsius (°C)** is also allowed in the SI system, with both the word "degree" and the degree symbol used for Celsius measurements. Celsius degrees are the same magnitude as those of kelvin, but the two scales place their zeros in different places. Water freezes at 273.15 K (0 °C) and boils at 373.15 K (100 °C) by definition, and normal human body temperature is approximately 310 K (37 °C). The conversion between these two units and the Fahrenheit scale will be discussed later in this chapter. The degree of precision varies when measuring temperature (Figure 2.1c).

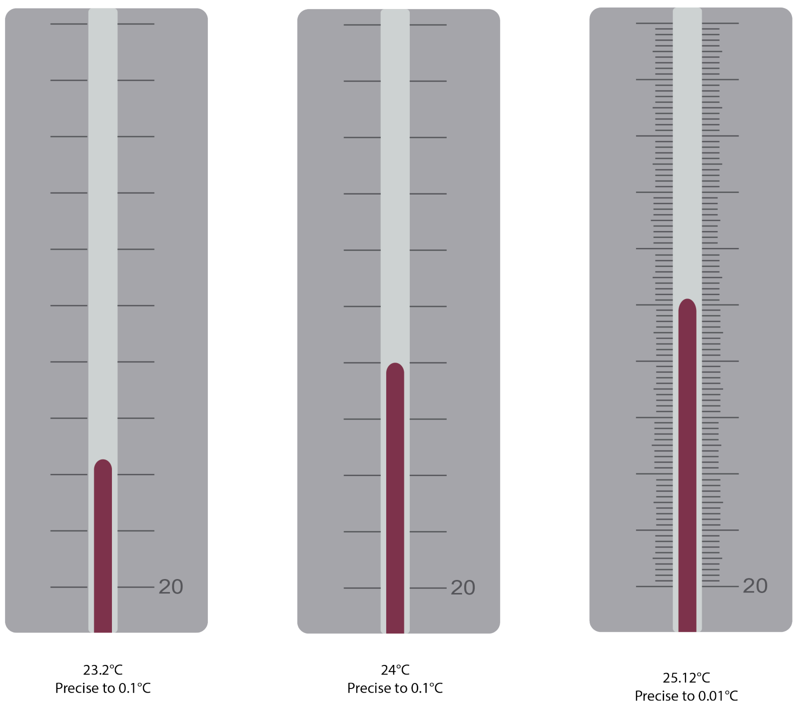

Figure 2.1c Different degrees of precision while measuring temperature. (credit: graphics by Revathi Mahadevan, CC BY 4.0)

Time

The SI base unit of time is the **second (s)**. Small and large time intervals can be expressed with the appropriate prefixes; for example, 3 microseconds = 0.000003 s = 3 × 10^{-6} and 5 megaseconds = 5,000,000 s = 5 × 10^{6} s. Alternatively, hours, days, and years can be used.

Derived SI Units

We can derive many units from the seven SI base units. For example, we can use the base unit of length to define a unit of volume, and the base units of mass and length to define a unit of density. Figures 2.1d and 2.1e demonstrate various volumes and masses of cubes.

Figure 2.1d The figure shows 3 cubes. Each of equal volume but with varying masses. (credit: graphics by Revathi Mahadevan, CC BY 4.0)

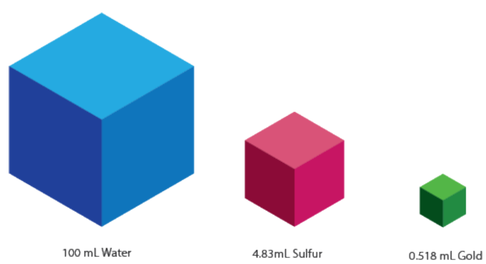

Figure 2.1e The figure shows 3 cubes. Each of equal masses but with varying volumes. (credit: graphics by Revathi Mahadevan, CC BY 4.0)

Volume

Volume is the measure of the amount of space occupied by an object. The standard SI unit of volume is defined by the base unit of length (Figure 2.1f). The standard volume is a **cubic meter** (m^3), a cube with an edge length of exactly one meter. To dispense a cubic meter of water, we could build a cubic box with edge lengths of exactly one meter. This box would hold a cubic meter of water or any other substance.

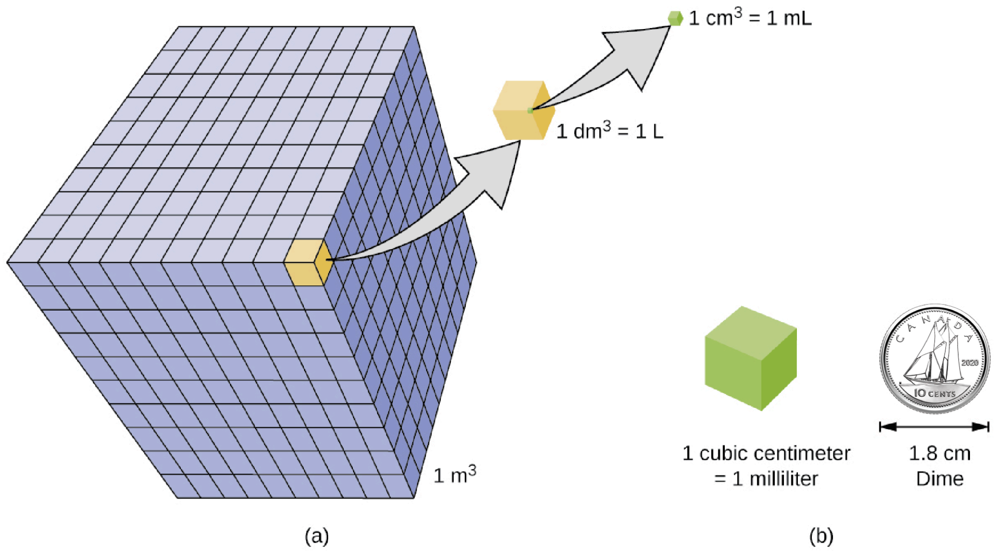

Figure 2.1f (a) The relative volumes are shown for cubes of 1 m^3, 1 dm^3 (1 L), and 1 cm^3 (1 mL) (not to scale). (b) The diameter of a dime is compared relative to the edge length of a 1-cm^3 (1-mL) cube. (credit: *Chemistry 2e (Open Stax)*, CC BY 4.0 . / Canadian dime added by Revathi Mahadevan.)

A more commonly used unit of volume is derived from the decimetre (0.1 m, or 10 cm). A cube with edge lengths of exactly one decimetre contains a volume of one cubic decimetre (dm^3). A **litre (L)** is the more common name for the cubic decimetre. One litre is about 1.06 quarts. A **cubic centimetre** (cm^3) is the volume of a cube with an edge length of exactly one centimetre. The abbreviation **cc** (for **c**ubic **c**entimetre) is often used by health professionals. A cubic centimetre is also called a **millilitre (mL)** and is 1/1000 of a litre.

Density

We use the mass and volume of a substance to determine its **density**. Thus, the units of density are defined by the base units of mass and length.

The density of a substance is the ratio of the mass of a sample of the substance to its volume. The SI unit for density is the kilogram per cubic meter (kg/m^3). For many situations, however, this as an inconvenient unit, and we often use grams per cubic centimetre (g/cm^3) for the densities of solids and liquids, and grams per litre (g/L) for gases. Although there are exceptions, most liquids and solids have densities that range from about 0.7 g/cm^3 (the density of gasoline) to 19 g/cm^3 (the density of gold). The density of air is about 1.2 g/L. Table 2.1c shows the densities of some common substances.

Table 2.1c Densities of Common Substances

Solids	**Liquids**	**Gases (at 25 °C and 1 atm)**
ice (at 0 °C) 0.92 g/cm^3	water 1.0 g/cm^3	dry air 1.20 g/L
oak (wood) 0.60–0.90 g/cm^3	ethanol 0.79 g/cm^3	oxygen 1.31 g/L
iron 7.9 g/cm^3	acetone 0.79 g/cm^3	nitrogen 1.14 g/L
copper 9.0 g/cm^3	glycerin 1.26 g/cm^3	carbon dioxide 1.80 g/L
lead 11.3 g/cm^3	olive oil 0.92 g/cm^3	helium 0.16 g/L
silver 10.5 g/cm^3	gasoline 0.70–0.77 g/cm^3	neon 0.83 g/L
gold 19.3 g/cm^3	mercury 13.6 g/cm^3	radon 9.1 g/L

While there are many ways to determine the density of an object, perhaps the most straightforward method involves separately finding the mass and volume of the object and then dividing the mass of the sample by its volume. In the following example, the mass is found directly by weighing, but the volume is found indirectly through length measurements.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Watch How taking a bath led to Archimedes' principle – Mark Salata (3 mins) (https://youtu.be/ijj58xD5fDI)

Example 2.1a

Density of lead

Calculation of Density Gold—in bricks, bars, and coins—has been a form of currency for centuries. In order to swindle people into paying for a brick of gold without actually investing in a brick of gold, people have considered filling the centres of hollow gold bricks with lead to fool buyers into thinking that the entire brick is gold. It does not work: Lead is a dense substance, but its density is not as great as that of gold, 19.3 g/cm^3. What is the density of lead if a cube of lead has an edge length of 2.00 cm and a mass of 90.7 g?

Solution

The density of a substance can be calculated by dividing its mass by its volume. The volume of a cube is calculated by cubing the edge length.

$$\text{volume of lead cube} = 2.00\text{cm} \times 2.00\text{cm} \times 2.00\text{cm} = 9.00\text{cm}^3$$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{90.7\text{g}}{8.00\text{cm}^3} = \frac{11.3\text{g}}{1.00\text{cm}^3} = 11.3 \text{ g/cm}^3$$

(We will discuss the reason for rounding to the first decimal place in the next section.)

Exercise 2.1a

1. To three decimal places, what is the volume of a cube (cm^3) with an edge length of 0.843 cm?
2. If the cube in part (a) is copper and has a mass of 5.34 g, what is the density of copper to two decimal places?

Check Your Answer[1]

Example 2.1b

Displacement of water to determine density.

The simulation uses the displacement of water to determine the density. In the Compare section, determine the density of the red and yellow blocks.

Practice using the following PhET simulation: Density (https://phet.colorado.edu/sims/html/density/latest/density_en.html)

Exercise 2.1b

Using the PhET Density simulation in Example 2.1b, remove all of the blocks from the water and add the green block to the tank of water, placing it approximately in the middle of the tank. Determine the density of the green block.

Check Your Answer[2]

Exercise 2.1c

Check Your Learning Exercise (Text Version)
For each of the following seven options (a-g), choose an SI base unit or derived unit from the word list that is appropriate for each given measurement:

Word List:

cubic millimeters, kilogram per cubic meter, degree Celsius, kilometer, meter per second, kilograms, square meters.

a. The mass of the moon
b. The distance from Vancouver to Toronto
c. The speed of sound
d. The density of air
e. The temperature at which alcohol boils
f. The area of the province of Newfoundland and Labrador
g. The volume of a flu shot or a measles vaccination

Check Your Answer[3]

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Links to Interactive Learning Tools

Explore various units of measurement and scientific notation in Measurement and Numbers (https://www.physicsclassroom.com/Concept-Builders/Chemistry/Measurement) from the Physics Classroom (https://www.physicsclassroom.com/).

Practice Scientific Notation Identification (https://h5pstudio.ecampusontario.ca/content/44305) from eCampusOntario H5P Studio (https://h5pstudio.ecampusontario.ca/).

Practice Density Calculation #1 (https://h5pstudio.ecampusontario.ca/content/44360) from eCampusOntario H5P Studio (https://h5pstudio.ecampusontario.ca/).

Practice Density Calculation #2 (https://h5pstudio.ecampusontario.ca/content/44343) from eCampusOntario H5P Studio (https://h5pstudio.ecampusontario.ca/).

Key Equations

- $\text{density} = \dfrac{\text{mass}}{\text{volume}}$

Attribution & References

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Notes

1. (1) 0.599cm^3; (2) 8.91 g/cm^3
2. 2.00 kg/L
3. (a) kilograms; (b) meters; (c) meters/second; (d) kilograms/cubic meter; (e) kelvin; (f) square meters; (g) cubic meters

2.2 MEASUREMENT UNCERTAINTY, ACCURACY, AND PRECISION

Learning Objectives

By the end of this section, you will be able to:

- Define accuracy and precision
- Differentiate between accuracy and precision
- Distinguish exact and uncertain numbers
- Correctly represent uncertainty in quantities using significant figures
- Apply proper rounding rules to computed quantities

Counting is the only type of measurement that is free from uncertainty, provided the number of objects being counted does not change while the counting process is underway. The result of such a counting measurement is an example of an **exact number**. If we count eggs in a carton, we know *exactly* how many eggs the carton contains. The numbers of defined quantities are also exact. By definition, 1 foot is exactly 12 inches, 1 inch is exactly 2.54 centimetres, and 1 gram is exactly 0.001 kilogram. Quantities derived from measurements other than counting, however, are uncertain to varying extents due to practical limitations of the measurement process used.

Significant Figures in Measurement

The numbers of measured quantities, unlike defined or directly counted quantities, are not exact. To measure the volume of liquid in a graduated cylinder, you should make a reading at the bottom of the meniscus, the lowest point on the curved surface of the liquid.

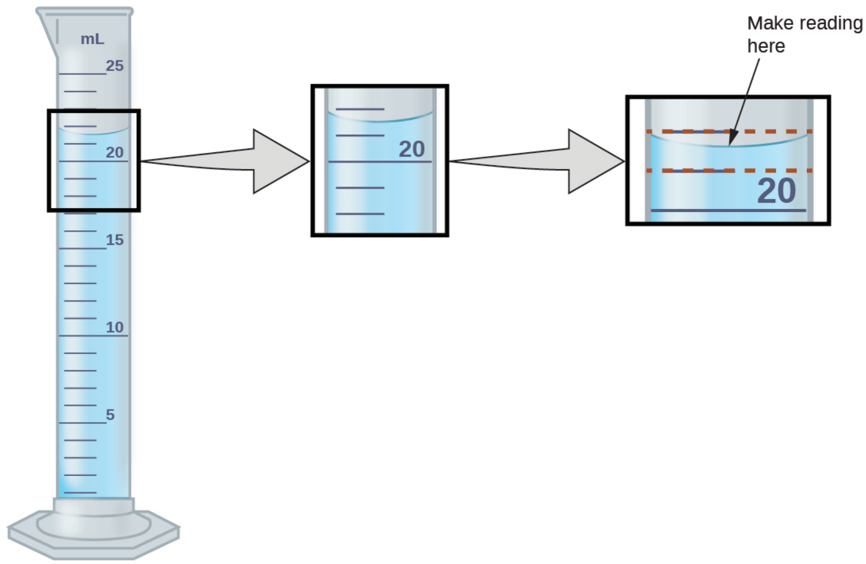


Figure 2.2a To measure the volume of liquid in this graduated cylinder, you must mentally subdivide the distance between the 21 and 22 mL marks into tenths of a millilitre, and then make a reading (estimate) at the bottom of the meniscus (credit: *Chemistry (OpenStax)*, CC BY 4.0).

Refer to the illustration in Figure 2.2a. The bottom of the meniscus in this case clearly lies between the 21 and 22 markings, meaning the liquid volume is *certainly* greater than 21 mL but less than 22 mL. The meniscus appears to be a bit closer to the 22-mL mark than to the 21-mL mark, and so a reasonable estimate of the liquid's volume would be 21.6 mL. In the number 21.6, then, the digits 2 and 1 are certain, but the 6 is an estimate. Some people might estimate the meniscus position to be equally distant from each of the markings and estimate the tenth-place digit as 5, while others may think it to be even closer to the 22-mL mark and estimate this digit to be 7. Note that it would be pointless to attempt to estimate a digit for the hundredths place, given that the tenths-place digit is uncertain. In general, numerical scales such as the one on this graduated cylinder will permit measurements to one-tenth of the smallest scale division. The scale in this case has 1-mL divisions, and so volumes may be measured to the nearest 0.1 mL.

This concept holds true for all measurements, even if you do not actively make an estimate. If you place a quarter on a standard electronic balance, you may obtain a reading of 6.72 g. The digits 6 and 7 are certain, and the 2 indicates that the mass of the quarter is likely between 6.71 and 6.73 grams. The quarter weighs *about* 6.72 grams, with a nominal uncertainty in the measurement of ± 0.01 gram. If we weigh the quarter on a more sensitive balance, we may find that its mass is 6.723 g. This means its mass lies between 6.722 and 6.724 grams, with an uncertainty of 0.001 grams. Every measurement has some **uncertainty**, which depends on the device used (and the user's ability). All of the digits in measurement, including the uncertain last digit, are called **significant figures** or **significant digits**. Note that zero may be a measured value; for example, if you stand on a scale that shows weight to the nearest pound and it shows "120," then the 1 (hundreds), 2 (tens), and 0 (ones) are all significant (measured) values.

Whenever you make a measurement properly, all the digits in the result are significant. But what if you were analyzing a reported value and trying to determine what is significant and what is not? Well, for starters, all nonzero digits are significant, and it is only zeros that require some thought. We will use the terms "leading," "trailing," and "captive" for the zeros and will consider how to deal with them (Figure 2.2b).

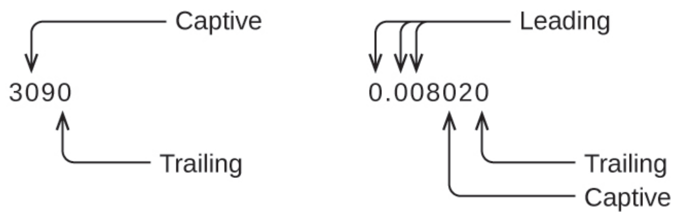


Figure 2.2b Examples of captive, leading and training zeros in multidigit numbers. A captive zero appears between two non-zero digitals. A leading zero appears before any non-zero digits. A trailing zero appears after the last non-zero digit (credit: *Chemistry (OpenStax)*, CC BY 4.0).

Starting with the first nonzero digit on the left, count this digit and all remaining digits to the right. This is the number of significant figures in the measurement unless the last digit is a trailing zero lying to the left of the decimal point (Figure 2.2c).

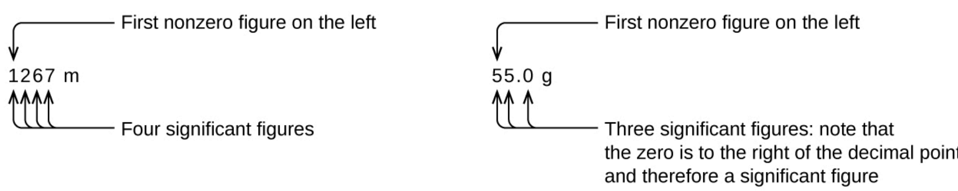


Figure 2.2c All non-zero digits are significant. Zeros to the right of a decimal point after a non-zero digit are significant (credit: *Chemistry (OpenStax)*, CC BY 4.0).

Captive zeros result from measurement and are therefore always significant. Leading zeros, however, are never significant—they merely tell us where the decimal point is located (Figure 2.2d).

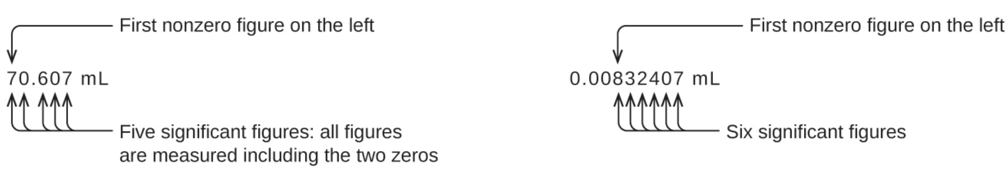


Figure 2.2d Trailing zeros may or may not be significant when there is no decimal. Assume the trailing zeros are not significant when there is no decimal (credit: *Chemistry (OpenStax)*, CC BY 4.0).

The leading zeros in this example are not significant. We could use the exponential notation (as described in Appendix B) and express the number as 8.32407×10^{-3}; then the number 8.32407 contains all of the significant figures, and 10^{-3} locates the decimal point.

The number of significant figures is uncertain in a number that ends with a zero to the left of the decimal point location. The zeros in the measurement 1,300 grams could be significant or they could simply indicate where the decimal point is located (Figure 2.2e). The ambiguity can be resolved with the use of exponential notation: 1.3×10^3 (two significant figures), 1.30×10^3 (three significant figures, if the tens place was measured), or 1.300×10^3 (four significant figures, if the one's place was also measured). In cases where only the decimal-formatted number is available, it is prudent to assume that all trailing zeros are not significant.

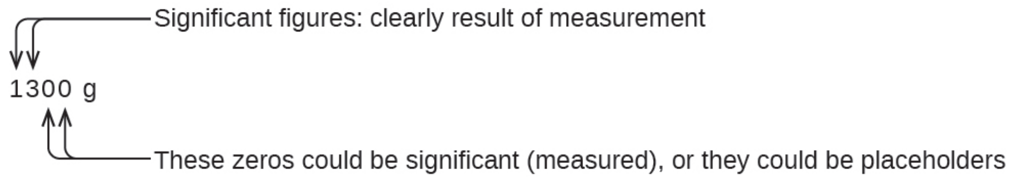


Figure 2.2e The zeros could be significant (measured) or placeholders depending on the measurement (credit: *Chemistry (OpenStax)*, CC BY 4.0).

When determining significant figures, be sure to pay attention to reported values and think about the measurement and significant figures in terms of what is reasonable or likely when evaluating whether the value makes sense. For example, the official January 2014 census reported the resident population of the US as 317,297,725. Do you think the US population was correctly determined to the reported nine significant figures, that is, to the exact number of people? People are constantly being born, dying, or moving into or out of the country, and assumptions are made to account for the large number of people who are not actually counted. Because of these uncertainties, it might be more reasonable to expect that we know the population to within perhaps a million or so, in which case the population should be reported as 3.17×10^8 people.

Significant Figures in Calculations

A second important principle of uncertainty is that results calculated from a measurement are at least as uncertain as the measurement itself. We must take the uncertainty in our measurements into account to avoid misrepresenting the uncertainty in calculated results. One way to do this is to report the result of a calculation with the correct number of significant figures, which is determined by the following three rules for **rounding** numbers:

1. When we add or subtract numbers, we should round the result to the same number of decimal places as the number with the least number of decimal places (the least precise value in terms of addition and subtraction).
2. When we multiply or divide numbers, we should round the result to the same number of digits as the number with the least number of significant figures (the least precise value in terms of multiplication and division).
3. If the digit to be dropped (the one immediately to the right of the digit to be retained) is less than 5, we

"round down" and leave the retained digit unchanged; if it is 5 or more, we "round up" and increase the retained digit by 1.

The following examples illustrate the application of this rule in rounding a few different numbers to three significant figures:

- 0.028675 rounds "up" to 0.0287 (the dropped digit, 7, and therefore round up)
- 18.3384 rounds "down" to 18.3 (the dropped digit, 3, and therefore round down)
- 6.8752 rounds "up" to 6.88 (the dropped digit is 5, and therefore round up)

Let's work through these rules with a few examples.

Example 2.2a

Rounding Numbers

Round the following to the indicated number of significant figures:

a. 31.57 (to two significant figures)
b. 8.1649 (to three significant figures)
c. 0.051065 (to four significant figures)
d. 0.90275 (to two significant figures)

Solution

a. 31.57 rounds "up" to 32 (the dropped digit is 5, and therefore round up)
b. 8.1649 rounds "down" to 8.16 (the dropped digit, 4, and therefore round down)
c. 0.051065 rounds "up" to 0.05107 (the dropped digit is 5, and therefore round up)
d. 0.90275 rounds "down" to 0.90 (the dropped digit is 2, and therefore round down)

Exercise 2.2a

Round the following to the indicated number of significant figures:

a. 0.424 (to two significant figures)
b. 0.0038661 (to three significant figures)
c. 421.25 (to four significant figures)
d. 28,683.5 (to five significant figures)

Check Your Answer[1]

Example 2.2b

Addition and Subtraction with Significant Figures

Rule: When we add or subtract numbers, we should round the result to the same number of decimal places as the number with the least number of decimal places (i.e., the least precise value in terms of addition and subtraction).

a. Add 1.0023 g and 4.383 g.
b. Subtract 421.23 g from 486 g.

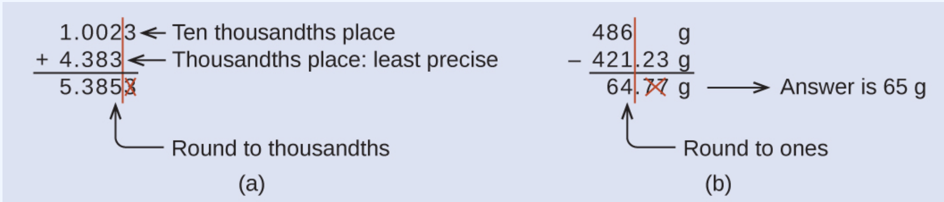
Solution

a.
$$\begin{array}{r} 1.0023\text{g} \\ +4.383\ \ \text{g} \\ \hline 5.3853\text{g} \end{array}$$

Answer is 5.385 g (round to the thousandths place; three decimal places)

b.
$$\begin{array}{r} 486\ \ \ \ \ \ \text{g} \\ -421.23\text{g} \\ \hline 64.77\text{g} \end{array}$$

Answer is 65 g (round to the ones place; no decimal places)



Exercise 2.2b

Correctly apply the rules of addition and subtraction with significant figures to perform the following calculations:

a. Add 2.334 mL and 0.31 mL.
b. Subtract 55.8752 m from 56.533 m.

Check Your Answer[2]

Example 2.2c

Multiplication and Division with Significant Figures

Rule: When we multiply or divide numbers, we should round the result to the same number of digits as the number with the least number of significant figures (the least precise value in terms of multiplication and division).

a. Multiply 0.6238 cm by 6.6 cm.
b. Divide 421.23 g by 486 mL.

Solution

a. 0.6238 cm × 6.6 cm = 4.11708 cm^2 → result is 4.1 cm^2 (round to two significant figures)
four significant figures × two significant figures → two significant figures answer

b. $\frac{421.23\text{g}}{486\text{mL}} = 0.86728\ldots\text{g/mL} \rightarrow \text{result is } 0.867\text{ g/mL (round to three significant figures)}$

$$\frac{\text{five significant figures}}{\text{three significant figures}} \rightarrow \text{three significant figures answer}$$

Exercise 2.2c

Correctly apply the rules of multiplication and division with significant figures to perform the following calculations:

a. Multiply 2.334 cm and 0.320 cm.
b. Divide 55.8752 m by 56.53 s.

Check Your Answer[3]

In the midst of all these technicalities, it is important to keep in mind the reason why we use significant figures and rounding rules—to correctly represent the certainty of the values we report and to ensure that a calculated result is not represented as being more certain than the least certain value used in the calculation.

Example 2.2d

Calculation with Significant Figures

One common bathtub is 13.44 dm long, 5.920 dm wide, and 2.54 dm deep. Assume that the tub is rectangular and calculate its approximate volume in liters.

Solution

$$\begin{aligned} V = \;& l \times w \times d \\ & 13.44\text{ dm} \times 5.920\text{ dm} \times 2.54\text{ dm} \\ & 202.09459\ldots\text{ dm}^3 \text{(value from calculator)} \\ & 202\text{ dm}^3,\text{ or } 202\text{ L (answer rounded to three significant figures)} \end{aligned}$$

Exercise 2.2d

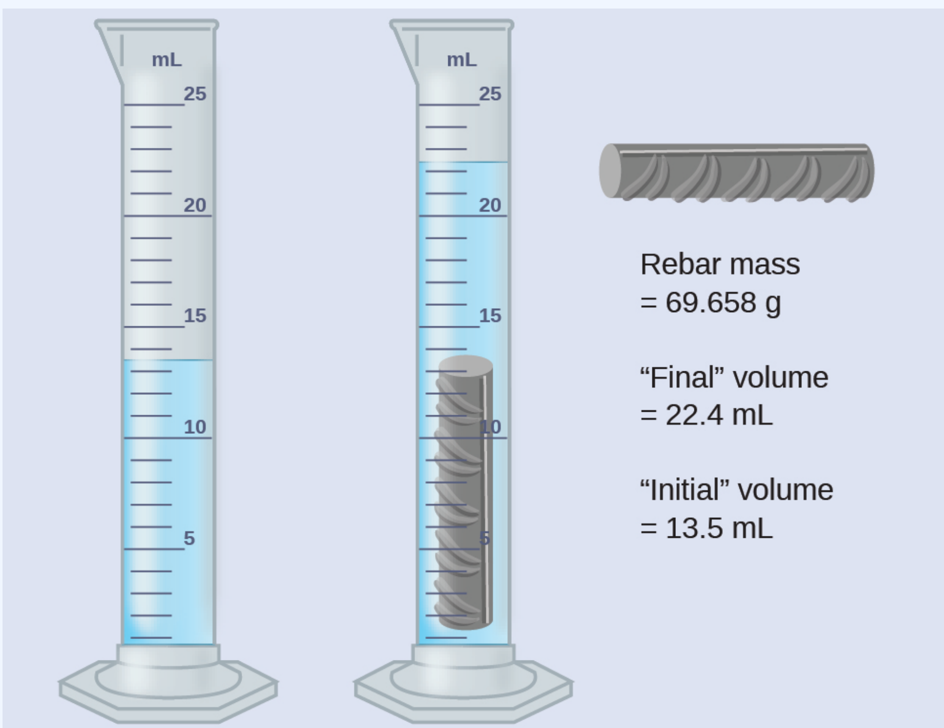
What is the density of a liquid with a mass of 31.1415 g and a volume of 30.13 cm^3?

Check Your Answer[4]

Example 2.2e

Experimental Determination of Density Using Water Displacement

A piece of rebar is weighed and then submerged in a graduated cylinder partially filled with water, with results as shown.



a. Use these values to determine the density of this piece of rebar.
b. Rebar is mostly iron. Does your result in (a) support this statement? How?

Solution

The volume of the piece of rebar is equal to the volume of the water displaced:

$$\text{volume} = 22.4\text{mL} - 13.5\text{mL} = 8.9\text{mL} = 8.9\text{cm}^3$$

(rounded to the nearest 0.1 mL, per the rule for addition and subtraction)

The density is the mass-to-volume ratio:

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{69.658\text{g}}{8.9\text{cm}^3} = 7.8\text{g}/\text{cm}^3$$

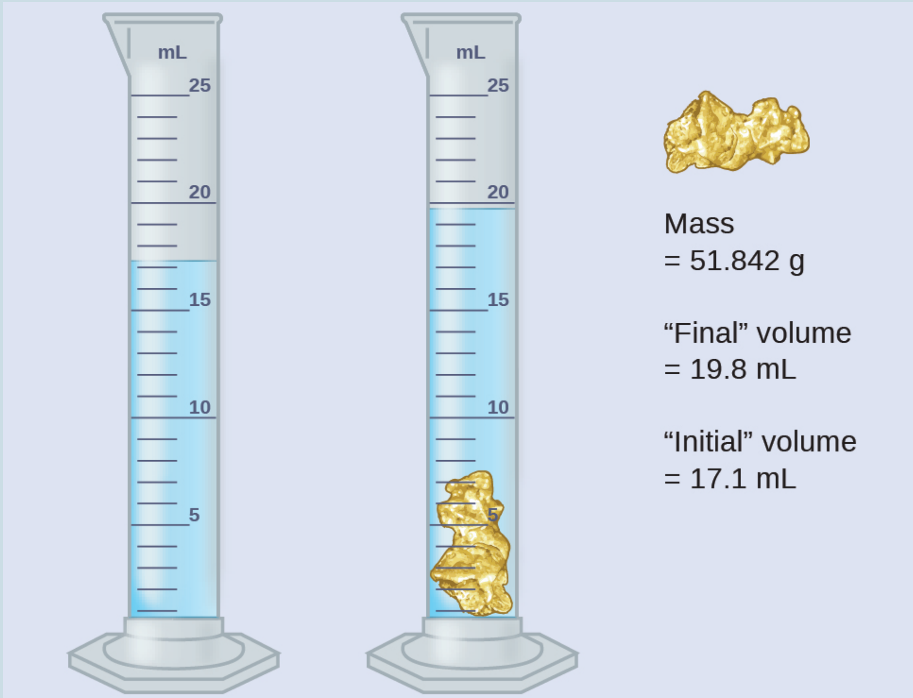
(rounded to two significant figures, per the rule for multiplication and division)

From Table 2.1c, the density of iron is 7.9 g/cm^3, very close to that of rebar, which lends some support to the fact that rebar is mostly iron.

Exercise 2.2e

Check Your Learning

An irregularly shaped piece of a shiny yellowish material is weighed and then submerged in a graduated cylinder, with results as shown.



a. Use these values to determine the density of this material.
b. Do you have any reasonable guesses as to the identity of this material? Explain your reasoning.

Check Your Answer[5]

Accuracy and Precision

Scientists typically make repeated measurements of a quantity to ensure the quality of their findings and to know both the **precision** and the **accuracy** of their results. Measurements are said to be precise if they yield very similar results when repeated in the same manner. A measurement is considered accurate if it yields a result that is very close to the true or accepted value. Precise values agree with each other; accurate values agree with a true value. These characterizations can be extended to other contexts, such as the results of an archery competition (Figure 2.2f).

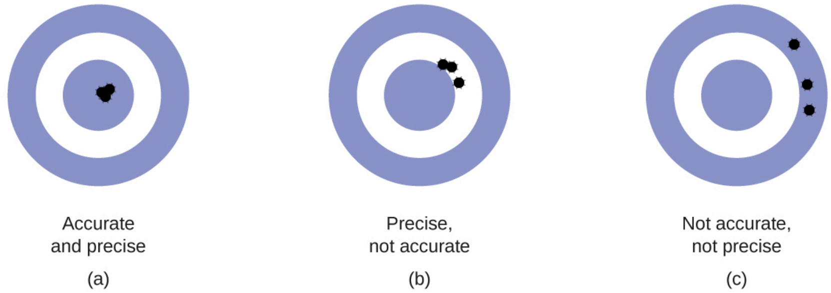


Figure 2.2f (a) These arrows are close to both the bull's eye and one another, so they are both accurate and precise. (b) These arrows are close to one another but not on target, so they are precise but not accurate. (c) These arrows are neither on target nor close to one another, so they are neither accurate nor precise (credit: *Chemistry (OpenStax)*, CC BY 4.0).

Suppose a quality control chemist at a pharmaceutical company is tasked with checking the accuracy and precision of three different machines that are meant to dispense 10 ounces (296 mL) of cough syrup into storage bottles. She proceeds to use each machine to fill five bottles and then carefully determines the actual volume dispensed, obtaining the results tabulated in Table 2.2a.

Table 2.2a Volume (mL) of Cough Medicine Delivered by 10-oz (296 mL) Dispensers

Dispenser #1	Dispenser #2	Dispenser #3
283.3	298.3	296.1
284.1	294.2	295.9
283.9	296.0	296.1
284.0	297.8	296.0
284.1	293.9	296.1

Considering these results, she will report that dispenser #1 is precise (values all close to one another, within a few tenths of a milliliter) but not accurate (none of the values are close to the target value of 296 mL, each being more than 10 mL too low). Results for dispenser #2 represent improved accuracy (each volume is less than 3 mL away from 296 mL) but worse precision (volumes vary by more than 4 mL). Finally, she can report that dispenser #3 is working well, dispensing cough syrup both accurately (all volumes within 0.1 mL of the target volume) and precisely (volumes differing from each other by no more than 0.2 mL).

Watch What's the difference between accuracy and precision? – Matt Anticole (5 mins). (https://www.youtube.com/watch?v=hRAFPdDppzs)

Links to Interactive Learning Tools

Explore significant digits in Significant Digits and Measurement (https://www.physicsclassroom.com/Concept-Builders/Chemistry/Significant-Digits) from the Physics Classroom (https://www.physicsclassroom.com/).

Attribution & References

Notes

1. (a) 0.42; (b) 0.00387; (c) 421.2; (d) 28,684
2. (a) 2.64 mL; (b) 0.658 m
3. (a) 0.747 cm^2 (b) 0.9884 m/s
4. 1.034 g/mL
5. (a) 19 g/cm^3; (b) It is likely gold; the right appearance for gold and very close to the density given for gold in Table 2.1c.

2.3 MATHEMATICAL TREATMENT OF MEASUREMENT RESULTS

Learning Objectives

By the end of this section, you will be able to:

- Explain the dimensional analysis (factor label) approach to mathematical calculations involving quantities
- Use dimensional analysis to carry out unit conversions for a given property and computations involving two or more properties

It is often the case that a quantity of interest may not be easy (or even possible) to measure directly but instead must be calculated from other directly measured properties and appropriate mathematical relationships. For example, consider measuring the average speed of an athlete running sprints. This is typically accomplished by measuring the *time* required for the athlete to run from the starting line to the finish line, and the *distance* between these two lines, and then computing *speed* from the equation that relates these three properties:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

An Olympic-quality sprinter can run 100 m in approximately 10 s, corresponding to an average speed of

$$\frac{100\text{ m}}{10\text{ s}} = 10\text{ m/s}$$

Note that this simple arithmetic involves dividing the numbers of each measured quantity to yield the number of the computed quantity (100/10 = 10) *and likewise* dividing the units of each measured quantity to yield the unit of the computed quantity (m/s = m/s). Now, consider using this same relation to predict the time required for a person running at this speed to travel a distance of 25 m. The same relation between the three properties is used, but in this case, the two quantities provided are a speed (10 m/s) and a distance (25 m). To yield the sought property, time, the equation must be rearranged appropriately:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

The time can then be computed as:

$$\frac{25\text{ m}}{10\text{ m/s}} = 2.5\text{ s}$$

Again, arithmetic on the numbers (25/10 = 2.5) was accompanied by the same arithmetic on the units (m/m/s = s) to yield the number and unit of the result, 2.5 s. Note that, just as for numbers, when a unit is divided by an identical unit (in this case, m/m), the result is "1"—or, as commonly phrased, the units "cancel."

These calculations are examples of a versatile mathematical approach known as **dimensional analysis** (or the **factor-label method**). Dimensional analysis is based on this premise: *the units of quantities must be subjected to the same mathematical operations as their associated numbers*. This method can be applied to computations ranging from simple unit conversions to more complex, multi-step calculations involving several different quantities.

Conversion Factors and Dimensional Analysis

A ratio of two equivalent quantities expressed with different measurement units can be used as a **unit conversion factor**. For example, the lengths of 2.54 cm and 1 in. are equivalent (by definition), and so a unit conversion factor may be derived from the ratio,

$$\frac{2.54\text{ cm}}{1\text{ in.}}\ (2.54\text{ cm} = 1\text{ in.})\text{ or }2.54\frac{\text{cm}}{\text{in}}$$

Several other commonly used conversion factors are given in Table 2.3a. A more complete list is available in Appendix C.

Table 2.3a Common Conversion Factors

Type of Unit	Unit	Equivalent Unit
length	1 m	1.0936 yd
length	1 in.	2.54 cm (exact)
length	1 km	0.62137 mi
length	1 mi	1609.3 m
volume	1 L	1.0567 qt
volume	1 qt	0.94635 L
volume	1 ft^3	28.317 L
volume	1 tbsp	14.787 mL
mass	1 kg	2.2046 lb
mass	1 lb	453.59 g
mass	1 (avoirdupois) oz	28.349 g
mass	1 (troy) oz	31.103 g

When we multiply a quantity (such as distance given in inches) by an appropriate unit conversion factor, we convert the quantity to an equivalent value with different units (such as distance in centimetres). For example, a basketball player's vertical jump of 34 inches can be converted to centimetres by:

$$34\text{ in.} \times \frac{2.54\text{ cm}}{1\ \cancel{\text{in}}} = 86\text{ cm}$$

Since this simple arithmetic involves *quantities*, the premise of dimensional analysis requires that we multiply both *numbers and units*. The numbers of these two quantities are multiplied to yield the number of the product quantity, 86, whereas the units are multiplied to yield $\frac{\text{in.} \times \text{cm}}{\text{in.}}$. Just as for numbers, a ratio of identical units is also numerically equal to one, $\frac{\text{in.}}{\text{in.}} = 1$, and the unit product thus simplifies to *cm*. (When identical units divide to yield a factor of 1, they are said to "cancel.") Using dimensional analysis, we can determine that a unit conversion factor has been set up correctly by checking to confirm that the original unit will cancel, and the result will contain the sought (converted) unit.

Example 2.3a

Using a Unit Conversion Factor

The mass of a competition frisbee is 125 g. Convert its mass to ounces using the unit conversion factor derived from the relationship 1 oz = 28.349 g (Table 2.3a).

Solution

If we have the conversion factor, we can determine the mass in kilograms using an equation similar the one used for converting length from inches to centimetres.

$$x \text{ oz} = 125 \text{ g} \times \text{unit conversion factor}$$

We write the unit conversion factor in its two forms:

$$\frac{1 \text{ oz}}{28.349 \text{ g}} \text{ and } \frac{28.349 \text{ g}}{1 \text{ oz}}$$

The correct unit conversion factor is the ratio that cancels the units of grams and leaves ounces.

$$x \text{ oz} = 125 \cancel{\text{g}} \times \frac{1 \text{ oz}}{28.349 \cancel{\text{g}}}$$

$$\left(\frac{125}{28.349}\right)\text{oz}$$

$$4.41 \text{ oz (three significant figures)}$$

Exercise 2.3a

Convert a volume of 9.345 qt to litres.

Check Your Answer[1]

Beyond simple unit conversions, the factor-label method can be used to solve more complex problems involving computations. Regardless of the details, the basic approach is the same—all the *factors* involved in the calculation must be appropriately oriented to insure that their *labels* (units) will appropriately cancel and/or combine to yield the desired unit in the result. This is why it is referred to as the factor-label method. As your study of chemistry continues, you will encounter many opportunities to apply this approach.

Example 2.3b

Computing Quantities from Measurement Results and Known Mathematical Relations

What is the density of common antifreeze in units of g/mL? A 4.00-qt sample of the antifreeze weighs 9.26 lb.

Solution

Since density = $\frac{\text{mass}}{\text{volume}}$, we need to divide the mass in grams by the volume in millilitres. In general: the number of units of B = the number of units of A × unit conversion factor. The necessary conversion factors are given in Table 2.3a: 1 lb = 453.59 g; 1 L = 1.0567 qt; 1 L = 1,000 mL. We can convert mass from pounds to grams in one step:

$$9.26\ \cancel{\text{lb}} \times \frac{453.59\ \text{g}}{1\ \cancel{\text{lb}}} = 4.20 \times 10^3\ \text{g}$$

We need to use two steps to convert volume from quarts to millilitres.

1. *Convert quarts to liters.*

$$4.00\ \cancel{\text{qt}} \times \frac{1\text{L}}{1.0567\cancel{\text{qt}}} = 3.78\text{L}$$

2. *Convert liters to milliliters.*

$$3.78\ \cancel{\text{L}} \times \frac{1000\ \text{L}}{\cancel{\text{L}}} = 3.78\ \text{L} \times 10^3\ \text{mL}$$

Then,

$$\text{density} = \frac{4.20 \times 10^3\ \text{g}}{3.78 \times 10^3\ \text{mL}} = 1.11\ \text{g/mL}$$

Alternatively, the calculation could be set up in a way that uses three unit conversion factors sequentially as follows:

$$\frac{9.26\cancel{\text{lb}}}{4.00\cancel{\text{qt}}} \times \frac{453.59\text{g}}{1\cancel{\text{L}}} \times \frac{1.0567\cancel{\text{qt}}}{1\cancel{\text{L}}} \times \frac{1\cancel{\text{L}}}{1000\text{mL}} = 1.11\text{g/mL}$$

Exercise 2.3b

What is the volume in litres of 1.000 oz, given that 1 L = 1.0567 qt and 1 qt = 32 oz (exactly)?

Check Your Answer[2]

Example 2.3c

Computing Quantities from Measurement Results and Known Mathematical Relations

While being driven from Philadelphia to Atlanta, a distance of about 1250 km, a 2014 Lamborghini Aventador Roadster uses 213 L gasoline.

- What (average) fuel economy, in miles per gallon, did the Roadster get during this trip?
- If gasoline costs \$3.80 per gallon, what was the fuel cost for this trip?

Solution

1. We first convert distance from kilometres to miles:
   $$1250\text{ km} \times \frac{0.62137\text{ mi}}{1\text{ km}} = 777\text{ mi}$$
   and then convert volume from liters to gallons:
   $$213\ \cancel{\text{L}} \times \frac{1.0567\ \cancel{\text{qt}}}{1\ \cancel{\text{L}}} \times \frac{1\text{ gal}}{4\ \cancel{\text{qt}}} = 56.3\text{ gal}$$
   Then,
   $$\text{(average) mileage} = \frac{777\text{ mi}}{56.3\text{ gal}} = 13.8\text{ miles/gallon} = 13.8\text{ mpg}$$
   Alternatively, the calculation could be set up in a way that uses all the conversion factors sequentially, as follows:
   $$\frac{1250\ \cancel{\text{km}}}{213\ \cancel{\text{L}}} \times \frac{0.62137\text{ mi}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{L}}}{1.0567\ \cancel{\text{qt}}} \times \frac{4\ \cancel{\text{qt}}}{1\text{ gal}} = 13.8\text{ mpg}$$
2. Using the previously calculated volume in gallons, we find:

$$56.3 \text{ gal} \times \frac{\$3.80}{1 \text{ gal}} = \$214$$

Exercise 2.3c

A Toyota Prius Hybrid uses 59.7 L gasoline to drive from San Francisco to Seattle, a distance of 1300 km (two significant digits).

1. What (average) fuel economy, in miles per gallon, did the Prius get during this trip?
2. If gasoline costs \$3.90 per gallon, what was the fuel cost for this trip?

Check Your Answer[3]

Conversion of Temperature Units

We use the word **temperature** to refer to the hotness or coldness of a substance. One way we measure a change in temperature is to use the fact that most substances expand when their temperature increases and contract when their temperature decreases. The mercury or alcohol in a common glass thermometer changes its volume as the temperature changes. Because the volume of the liquid changes more than the volume of the glass, we can see the liquid expand when it gets warmer and contract when it gets cooler.

To mark a scale on a thermometer, we need a set of reference values: Two of the most commonly used are the freezing and boiling temperatures of water at a specified atmospheric pressure. On the Celsius scale, 0 °C is defined as the freezing temperature of water and 100 °C as the boiling temperature of water. The space between the two temperatures is divided into 100 equal intervals, which we call degrees. On the **Fahrenheit** scale, the freezing point of water is defined as 32 °F and the boiling temperature as 212 °F. The space between these two points on a Fahrenheit thermometer is divided into 180 equal parts (degrees).

Defining the Celsius and Fahrenheit temperature scales as described in the previous paragraph results in a slightly more complex relationship between temperature values on these two scales than for different units of measure for other properties. Most measurement units for a given property are directly proportional to one another (y = mx). Using familiar length units as one example:

$$\text{length in feet} = \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) \times \text{length in inches}$$

where y = length in feet, x = length in inches, and the proportionality constant, m, is the conversion factor. The Celsius and Fahrenheit temperature scales, however, do not share a common zero point, and so the relationship between these two scales is a linear one rather than a proportional one (y = mx + b). Consequently, converting a temperature from one of these scales into the other requires more than simple multiplication by a conversion factor, m, it also must take into account differences in the scales' zero points (b).

The linear equation relating Celsius and Fahrenheit temperatures is easily derived from the two temperatures used to define each scale. Representing the Celsius temperature as x and the Fahrenheit temperature as y, the slope, m, is computed to be:

$$m = \frac{\Delta y}{\Delta x} = \frac{212\ ^\circ\text{F} - 32\ ^\circ\text{F}}{100\ ^\circ\text{C} - 0\ ^\circ\text{C}} = \frac{180\ ^\circ\text{F}}{100\ ^\circ\text{C}} = \frac{9\ ^\circ\text{F}}{5\ ^\circ\text{C}}$$

The y-intercept of the equation, b, is then calculated using either of the equivalent temperature pairs, (100 °C, 212 °F) or (0 °C, 32 °F), as:

$$b = y - mx = 32\ ^\circ\text{F} - \frac{9\ ^\circ\text{F}}{5\ ^\circ\text{C}} \times 0\ ^\circ\text{C} = 32\ ^\circ\text{F}$$

The equation relating the temperature scales is then:

$$T_{^\circ\text{F}} = (\frac{9\ ^\circ\text{F}}{5\ ^\circ\text{C}} \times T_{^\circ\text{C}}) + 32\ ^\circ\text{C}$$

An abbreviated form of this equation that omits the measurement units is:

$$T_{^\circ\text{F}} = \frac{9}{5} \times T_{^\circ\text{C}} + 32$$

Rearrangement of this equation yields the form useful for converting from Fahrenheit to Celsius:

$$T_{^\circ\text{C}} = \frac{5}{9}(T_{^\circ\text{F}} - 32)$$

As mentioned earlier in this chapter, the SI unit of temperature is the kelvin (K). Unlike the Celsius and Fahrenheit scales, the kelvin scale is an absolute temperature scale in which 0 (zero) K corresponds to the lowest temperature that can theoretically be achieved. The early 19th-century discovery of the relationship between a gas's volume and temperature suggested that the volume of a gas would be zero at −273.15 °C. In 1848, British physicist William Thompson, who later adopted the title of Lord Kelvin, proposed an absolute temperature scale based on this concept (further treatment of this topic is provided in this text's chapter on gases).

The freezing temperature of water on this scale is 273.15 K and its boiling temperature 373.15 K. Notice the numerical difference in these two reference temperatures is 100, the same as for the Celsius scale, and so the linear relation between these two temperature scales will exhibit a slope of $1\ \frac{\text{K}}{^\circ\text{C}}$. Following the same approach, the equations for converting between the kelvin and Celsius temperature scales are derived to be:

$$T_{\text{K}} = T_{^\circ\text{C}} + 273.15$$
$$T_{^\circ\text{C}} = T_{\text{K}} - 273.15$$

The 273.15 in these equations has been determined experimentally, so it is not exact. Figure 2.3a shows the relationship between the three temperature scales. Recall that we do not use the degree sign with temperatures on the Kelvin scale.

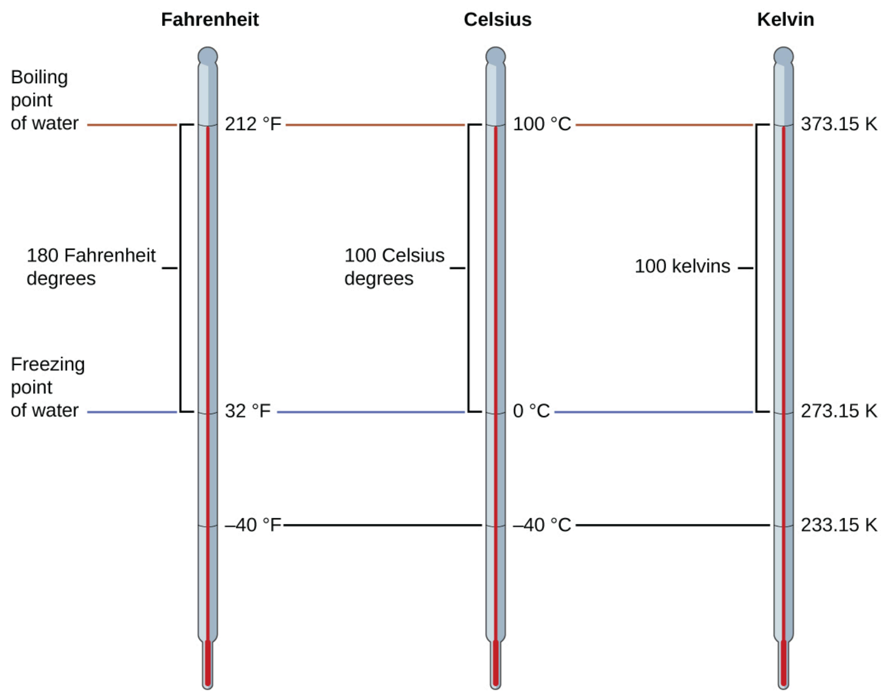


Figure 2.3a The Fahrenheit, Celsius, and kelvin temperature scales are compared.

Although the kelvin (absolute) temperature scale is the official SI temperature scale, Celsius is commonly used in many scientific contexts and is the scale of choice for nonscience contexts in almost all areas of the world. Very few countries (the U.S. and its territories, the Bahamas, Belize, Cayman Islands, and Palau) still use Fahrenheit for weather, medicine, and cooking.

Example 2.3d

Conversion from Celsius

Normal body temperature has been commonly accepted as 37.0 °C (although it varies depending on time of day and method of measurement, as well as among individuals). What is this temperature on the kelvin scale and on the Fahrenheit scale?

Solution

$$\mathrm{K} = ^\circ\mathrm{C} + 273.15 = 37.0 + 273.2 = 310.2\ \mathrm{K}$$
$$^\circ\mathrm{F} = \frac{9}{5}{}^\circ\mathrm{C} + 32.0 = (\frac{9}{5} \times 37.0) + 32.0 = 66.6 + 32.0 = 98.6^\circ\mathrm{F}$$

Exercise 2.3d

Convert 80.92 °C to K and °F.

Check Your Answer[4]

Example 2.3e

Conversion from Fahrenheit

Baking a ready-made pizza calls for an oven temperature of 450 °F. If you are in Europe, and your oven thermometer uses the Celsius scale, what is the setting? What is the kelvin temperature?

Solution

In Celsius:

$$\begin{gathered} {}^\circ\mathrm{C} = \frac{5}{9}({}^\circ\mathrm{F} - 32) = \frac{5}{9}(450 - 32) = \frac{5}{9} \times 418 = 232\ {}^\circ\mathrm{C} \\ \longrightarrow \text{set oven to } 230\ {}^\circ\mathrm{C}(\text{two significant figures}) \end{gathered}$$

In kelvin:

$$\begin{gathered} \mathrm{K} = {}^\circ\mathrm{C} + 273.15 = 230 + 273 = 503\ \mathrm{K} \\ \longrightarrow 5.0 \times 10^2\,\mathrm{K}(\text{two significant figures}) \end{gathered}$$

Exercise 2.3e

Convert 50 °F to °C and K.

Check Your Answer[5]

Links to Interactive Learning Tools

Explore the Metric System (https://www.physicsclassroom.com/Concept-Builders/Chemistry/Metric-System)from the Physics Classroom (https://www.physicsclassroom.com/).

Explore Metric Conversions (https://www.physicsclassroom.com/Concept-Builders/Chemistry/Metric-Conversions)from the Physics Classroom (https://www.physicsclassroom.com/).

Key Equations

- $T_{^\circ\mathrm{C}} = \dfrac{5}{9} \times T_{^\circ\mathrm{F}} - 32$
- $T_{^\circ\mathrm{F}} = \dfrac{9}{5} \times T_{^\circ\mathrm{C}} + 32$
- $T_{\mathrm{K}} = {}^\circ\mathrm{C} + 273.15$
- $T_{^\circ\mathrm{C}} = \mathrm{K} - 273.15$

Attributions & References

Notes

1. 8.844 L
2. 2.956 × 10^{-2}L
3. (a) 51 mpg; (b) $62
4. 354.07 K, 177.7 °F
5. 10 °C, 280 K

CHAPTER 2 - SUMMARY

2.1 Measurements

Measurements provide quantitative information that is critical in studying and practicing chemistry. Each measurement has an amount, a unit for comparison, and an uncertainty. Measurements can be represented in either decimal or scientific notation. Scientists primarily use the SI (International System) or metric systems. We use base SI units such as meters, seconds, and kilograms, as well as derived units, such as litres (for volume) and g/cm^3 (for density). In many cases, we find it convenient to use unit prefixes that yield fractional and multiple units, such as microseconds (10^{-6} seconds) and megahertz (10^6 hertz), respectively.

2.2 Measurement Uncertainty, Accuracy, and Precision

Quantities can be exact or measured. Measured quantities have an associated uncertainty that is represented by the number of significant figures in the measurement. The uncertainty of a calculated value depends on the uncertainties in the values used in the calculation and is reflected in how the value is rounded. Measured values can be accurate (close to the true value) and/or precise (showing little variation when measured repeatedly).

2.3 Mathematical Treatment of Measurement Results

Measurements are made using a variety of units. It is often useful or necessary to convert a measured quantity from one unit into another. These conversions are accomplished using unit conversion factors, which are derived by simple applications of a mathematical approach called the factor-label method or dimensional analysis. This strategy is also employed to calculate sought quantities using measured quantities and appropriate mathematical relations.

Attributions & References

Except where otherwise noted, this page is adapted by JR van Haarlem from "1.4 Measurements", "1.5 Measurement Uncertainty, Accuracy, and Precision" and "1.6 Mathematical Treatment of Measurement Results" In *General Chemistry 1 & 2* (https://boisestate.pressbooks.pub/chemistry/) by Rice University, a derivative of *Chemistry (Open Stax)* by Paul Flowers, Klaus Theopold, Richard Langley & William R.

CHAPTER 2 - REVIEW

2.1 Measurements

1. Is one liter about an ounce, a pint, a quart, or a gallon?
2. Is a meter about an inch, a foot, a yard, or a mile?
   Check answers: [1]
3. Indicate the SI base units or derived units that are appropriate for the following measurements:
   a. the length of a marathon race (26 miles 385 yards)
   b. the mass of an automobile
   c. the volume of a swimming pool
   d. the speed of an airplane
   e. the density of gold
   f. the area of a football field
   g. the maximum temperature at the South Pole on April 1, 1913
4. Indicate the SI base units or derived units that are appropriate for the following measurements:
   a. the mass of the moon
   b. the distance from Dallas to Oklahoma City
   c. the speed of sound
   d. the density of air
   e. the temperature at which alcohol boils
   f. the area of the state of Delaware
   g. the volume of a flu shot or a measles vaccination
      Check answers: [2]
5. Give the name and symbol of the prefixes used with SI units to indicate multiplication by the following exact quantities.
   a. 10^3
   b. 10^{-2}
   c. 0.1
   d. 10^{-3}
   e. 1,000,000
   f. 0.000001
6. Give the name of the prefix and the quantity indicated by the following symbols that are used with SI base units.

a. c
b. d
c. G
d. k
e. m
f. n
g. p
h. T

Check answers: [3]

7. A large piece of jewelry has a mass of 132.6 g. A graduated cylinder initially contains 48.6 mL water. When the jewelry is submerged in the graduated cylinder, the total volume increases to 61.2 mL.
   a. Determine the density of this piece of jewelry.
   b. Assuming that the jewelry is made from only one substance, what substance is it likely to be? Explain.
8. Visit this density simulation (https://www.simbucket.com/density/) and click the “turn fluid into water” button to adjust the density of liquid in the beaker to 1.00 g/mL.
   a. Use the water displacement approach to measure the mass and volume of the unknown material (select the green block with question marks).
   b. Use the measured mass and volume data from step (a) to calculate the density of the unknown material.
   c. Link out to the link provided.
   d. Assuming this material is a copper-containing gemstone, identify its three most likely identities by comparing the measured density to the values tabulated in this gemstone density guide.
   e. How are mass and density related for blocks of the same volume?

   Check answers: [4]
9. Visit this density simulation (https://www.simbucket.com/density/) and click the “reset” button to ensure all simulator parameters are at their default values.
   a. Use the water displacement approach to measure the mass and volume of the red block.
   b. Use the measured mass and volume data from step (a) to calculate the density of the red block.
   c. Use the vertical green slide control to adjust the fluid density to values well above, then well below, and finally nearly equal to the density of the red block, reporting your observations.
10. Visit this density simulation (https://www.simbucket.com/density/) and click the “turn fluid into water” button to adjust the density of liquid in the beaker to 1.00 g/mL. Change the block material to foam, and then wait patiently until the foam block stops bobbing up and down in the water.
    a. The foam block should be floating on the surface of the water (that is, only partially submerged). What is the volume of water displaced?

b. Use the water volume from part (a) and the density of water (1.00 g/mL) to calculate the mass of water displaced.
c. Remove and weigh the foam block. How does the block's mass compare to the mass of displaced water from part (b)? **Check answers:** [5]

2.2 Measurement Uncertainty, Accuracy, and Precision

1. Express each of the following numbers in scientific notation with correct significant figures:
   a. 711.0
   b. 0.03344
   c. 547.9
   d. 22086
   e. 1000.00
   f. 0.0000000651
   g. 0.007157
2. Express each of the following numbers in exponential notation with correct significant figures:
   a. 704
   b. 0.03344
   c. 547.9
   d. 22086
   e. 1000.00
   f. 0.0000000651
   g. 0.007157

   Check answers: [6]
3. Indicate whether each of the following can be determined exactly or must be measured with some degree of uncertainty:
   a. the number of eggs in a basket
   b. the mass of a dozen eggs
   c. the number of gallons of gasoline necessary to fill an automobile gas tank
   d. the number of cm in 2 m
   e. the mass of a textbook
   f. the time required to drive from San Francisco to Kansas City at an average speed of 53 mi/h
4. Indicate whether each of the following can be determined exactly or must be measured with some degree of uncertainty:
   a. the number of seconds in an hour
   b. the number of pages in this book
   c. the number of grams in your weight

d. the number of grams in 3 kilograms
e. the volume of water you drink in one day
f. the distance from San Francisco to Kansas City

Check answers: [7]

5. How many significant figures are contained in each of the following measurements?
   a. 38.7 g
   b. 2×10^{18} m
   c. 3,486,002 kg
   d. 9.74150×10^{-4} J
   e. 0.0613 cm^3
   f. 17.0 kg
   g. 0.01400 g/mL
6. How many significant figures are contained in each of the following measurements?
   a. 53 cm
   b. 2.05×10^{8} m
   c. 86,002 J
   d. 9.740×10^{4} m/s
   e. 10.0613 m^3
   f. 0.17 g/mL
   g. 0.88400 s

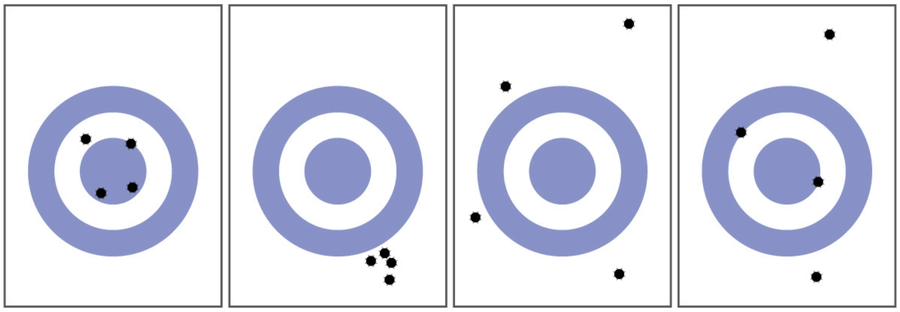
   Check answers: [8]
7. The following quantities were reported on the labels of commercial products. Determine the number of significant figures in each.
   a. 0.0055 g active ingredients
   b. 12 tablets
   c. 3% hydrogen peroxide
   d. 5.5 ounces
   e. 473 mL
   f. 1.75% bismuth
   g. 0.001% phosphoric acid
   h. 99.80% inert ingredients
8. Round off each of the following numbers to two significant figures:
   a. 0.436
   b. 9.000
   c. 27.2
   d. 135
   e. 1.497×10^{-3}

f. 0.445

Check answers: [9]

9. Round off each of the following numbers to two significant figures:
   a. 517
   b. 86.3
   c. 6.382×10^3
   d. 5.0008
   e. 22.497
   f. 0.885
10. Perform the following calculations and report each answer with the correct number of significant figures.
    a. 628×342
    b. $(5.63 \times 10^2) \times (7.4 \times 10^3)$
    c. $\frac{28.0}{13.483}$
    d. 8119×0.000023
    e. $14.98 + 27{,}340 + 84.7593$
    f. $42.7 + 0.259$

    Check answers: [10]
11. Perform the following calculations and report each answer with the correct number of significant figures.
    a. 62.8×34
    b. $0.147 + 0.0066 + 0.012$
    c. $38 \times 95 \times 1.792$
    d. $15 – 0.15 – 0.6155$
    e. $8.78 \times (\frac{0.0500}{0.478})$
    f. $140 + 7.68 + 0.014$
    g. $28.7 – 0.0483$
    h. $\frac{(88.5 - 87.57)}{45.13}$
12. Consider the results of the archery contest shown in this figure.
    a. Which archer is most precise?
    b. Which archer is most accurate?
    c. Who is both least precise and least accurate?



Check answers: [11]

13. Classify the following sets of measurements as accurate, precise, both, or neither.
    a. Checking for consistency in the weight of chocolate chip cookies: 17.27 g, 13.05 g, 19.46 g, 16.92 g
    b. Testing the volume of a batch of 25-mL pipettes: 27.02 mL, 26.99 mL, 26.97 mL, 27.01 mL
    c. Determining the purity of gold: 99.9999%, 99.9998%, 99.9998%, 99.9999%

2.3 Mathematical Treatment of Measurement Results

1. Write conversion factors (as ratios) for the number of:
    a. yards in 1 meter
    b. liters in 1 liquid quart
    c. pounds in 1 kilogram

    Check answers: [12]
2. Write conversion factors (as ratios) for the number of:
    a. kilometres in 1 mile
    b. litres in 1 cubic foot
    c. grams in 1 ounce
3. The label on a soft drink bottle gives the volume in two units: 2.0 L and 67.6 fl oz. Use this information to derive a conversion factor between the English and metric units. How many significant figures can you justify in your conversion factor?
   Check answers: [13]
4. The label on a box of cereal gives the mass of cereal in two units: 978 grams and 34.5 oz. Use this information to find a conversion factor between the English and metric units. How many significant figures can you justify in your conversion factor?
5. Soccer is played with a round ball having a circumference between 27 and 28 in. and a weight between 14 and 16 oz. What are these specifications in units of centimetres and grams?
   Check answers: [14]
6. A woman's basketball has a circumference between 28.5 and 29.0 inches and a maximum weight of 20 ounces (two significant figures). What are these specifications in units of centimetres and grams?

7. How many millilitres of a soft drink is contained in a 12.0-oz can?
   Check answers: [15]
8. A barrel of oil is exactly 42 gal. How many litres of oil are in a barrel?
9. The diameter of a red blood cell is about 3×10^{-4} in. What is its diameter in centimetres?
   Check answers: [16]
10. The distance between the centres of the two oxygen atoms in an oxygen molecule is 1.21×10^{-8} cm. What is this distance in inches?
11. Is a 197-lb weight lifter light enough to compete in a class limited to those weighing 90 kg or less?
    Check answers: [17]
12. A very good 197-lb weight lifter lifted 192 kg in a move called the clean and jerk. What was the mass of the weight lifted in pounds?
13. Many medical laboratory tests are run using 5.0 μL blood serum. What is this volume in millilitres?
    Check answers: [18]
14. If an aspirin tablet contains 325 mg aspirin, how many grams of aspirin does it contain?
15. Use scientific (exponential) notation to express the following quantities in terms of the SI base units in Table 2.1c in Chapter 2.1 Measurements:
    1. 0.13 g
    2. 232 Gg
    3. 5.23 pm
    4. 86.3 mg
    5. 37.6 cm
    6. 54 μm
    7. 1 Ts
    8. 27 ps
    9. 0.15 mK

       Check answers: [19]
16. Complete the following conversions between SI units.
    a. 612 g = ________ mg
    b. 8.160 m = ________ cm
    c. 3779 μg = ________ g
    d. 781 mL = ________ L
    e. 4.18 kg = ________ g
    f. 27.8 m = ________ km
    g. 0.13 mL = ________ L
    h. 1738 km = ________ m
    i. 1.9 Gg = ________ g
17. Gasoline is sold by the litre in many countries. How many litres are required to fill a 12.0-gal gas tank?

Check answers: [20]

18. Milk is sold by the litre in many countries. What is the volume of exactly 1/2 gal of milk in litres?
19. A long ton is defined as exactly 2240 lb. What is this mass in kilograms?

    Check answers: [21]

20. Make the conversion indicated in each of the following:
    a. the men's world record long jump, 29 ft 4¼ in., to meters
    b. the greatest depth of the ocean, about 6.5 mi, to kilometres
    c. the area of the state of Oregon, 96,981 mi^2, to square kilometres
    d. the volume of 1 gill (exactly 4 oz) to millilitres
    e. the estimated volume of the oceans, 330,000,000 mi^3, to cubic kilometres.
    f. the mass of a 3525-lb car to kilograms
    g. the mass of a 2.3-oz egg to grams
21. Make the conversion indicated in each of the following:
    a. the length of a soccer field, 120 m (three significant figures), to feet
    b. the height of Mt. Kilimanjaro, at 19,565 ft the highest mountain in Africa, to kilometres
    c. the area of an 8.5 t 11-inch sheet of paper in cm^2
    d. the displacement volume of an automobile engine, 161 $in.^3$, to litres
    e. the estimated mass of the atmosphere, 5.6 t 10^{15} tons, to kilograms
    f. the mass of a bushel of rye, 32.0 lb, to kilograms
    g. the mass of a 5.00-grain aspirin tablet to milligrams (1 grain = 0.00229 oz)

       Check answers: [22]

22. Many chemistry conferences have held a 50-Trillion Angstrom Run (two significant figures). How long is this run in kilometres and in miles? (1 Å = 1×10^{-10} m)
23. A chemist's 50-Trillion Angstrom Run would be an archeologist's 10,900 cubit run. How long is one cubit in meters and in feet? (1 Å = 1×10^{-8} cm)

    Check answers: [23]

24. The gas tank of a certain luxury automobile holds 22.3 gallons according to the owner's manual. If the density of gasoline is 0.8206 g/mL, determine the mass in kilograms and pounds of the fuel in a full tank.
25. As an instructor is preparing for an experiment, he requires 225 g phosphoric acid. The only container readily available is a 150-mL Erlenmeyer flask. Is it large enough to contain the acid, whose density is 1.83 g/mL?

    Check answers: [24]

26. To prepare for a laboratory period, a student lab assistant needs 125 g of a compound. A bottle containing 1/4 lb is available. Did the student have enough of the compound?
27. A chemistry student is 159 cm tall and weighs 45.8 kg. What is her height in inches and weight in pounds?

Check answers: [25]

28. In a recent Grand Prix, the winner completed the race with an average speed of 229.8 km/h. What was his speed in miles per hour, meters per second, and feet per second?
29. Solve these problems about lumber dimensions.
    a. To describe to a European how houses are constructed in the US, the dimensions of "two-by-four" lumber must be converted into metric units. The thickness × width × length dimensions are 1.50 in. × 3.50 in. × 8.00 ft in the US. What are the dimensions in cm × cm × m?
    b. This lumber can be used as vertical studs, which are typically placed 16.0 in. apart. What is that distance in centimetres?

    Check answers: [26]
30. The mercury content of a stream was believed to be above the minimum considered safe—1 part per billion (ppb) by weight. An analysis indicated that the concentration was 0.68 parts per billion. What quantity of mercury in grams was present in 15.0 L of the water, the density of which is 0.998 g/ml? ($1\ \text{ppb}\backslash;\text{Hg} = \dfrac{1\ \text{ng Hg}}{1\ \text{g water}}$)
31. Calculate the density of aluminum if 27.6 cm^3 has a mass of 74.6 g.

    Check answers: [27]
32. Osmium is one of the densest elements known. What is its density if 2.72 g has a volume of 0.121 cm^3?
33. Calculate these masses.
    a. What is the mass of 6.00 cm^3 of mercury, density = 13.5939 g/cm^3?
    b. What is the mass of 25.0 mL octane, density = 0.702 g/cm^3?

    Check answers: [28]
34. Calculate these masses.
    a. What is the mass of 4.00 cm^3 of sodium, density = 0.97 g/cm^3 ?
    b. What is the mass of 125 mL gaseous chlorine, density = 3.16 g/L?
35. Calculate these volumes.
    a. What is the volume of 25 g iodine, density = 4.93 g/cm^3?
    b. What is the volume of 3.28 g gaseous hydrogen, density = 0.089 g/L?

    Check answers: [29]
36. Calculate these volumes.
    a. What is the volume of 11.3 g graphite, density = 2.25 g/cm^3?
    b. What is the volume of 39.657 g bromine, density = 2.928 g/cm^3?
37. Convert the boiling temperature of gold, 2966 °C, into degrees Fahrenheit and kelvin.

    Check answers: [30]
38. Convert the temperature of scalding water, 54 °C, into degrees Fahrenheit and kelvin.
39. Convert the temperature of the coldest area in a freezer, −10 °F, to degrees Celsius and kelvin.

    Check answers: [31]

40. Convert the temperature of dry ice, −77 °C, into degrees Fahrenheit and kelvin.
41. Convert the boiling temperature of liquid ammonia, −28.1 °F, into degrees Celsius and kelvin.
    Check answers: [32]
42. The label on a pressurized can of spray disinfectant warns against heating the can above 130 °F. What are the corresponding temperatures on the Celsius and kelvin temperature scales?
43. The weather in Europe was unusually warm during the summer of 1995. The TV news reported temperatures as high as 45 °C. What was the temperature on the Fahrenheit scale?
    Check answers: [33]

Attribution & References

Notes

1. about a yard
2. (a) kilograms; (b) meters; (c) meters/second; (d) kilograms/cubic meter; (e) kelvin; (f) square meters; (g) cubic meters
3. (a) centi-, X 10^{-2}; (b) deci-, X 10^{-1}; (c) Giga-, X 10^{9}; (d) kilo-, X 10^{3}; (e) milli-, X 10^{-3}; (f) nano-, X 10^{-9}; (g) pico-, X 10^{-12}; (h) tera-, X 10^{12}
4. (a) m = 18.58 g, V = 5.7 mL. (b) d = 3.3 g/mL (c) dioptase (copper cyclosilicate, d = 3.28—3.31 g/mL); malachite (basic copper carbonate, d = 3.25—4.10 g/mL); Paraiba tourmaline (sodium lithium boron silicate with copper, d = 2.82—3.32 g/mL)
5. (a) $V_{\text{without block}}$= 25.5 mL, $V_{\text{with block}}$=28.3 mL, $V_{\text{displaced}}$=2.8 mL (b) m_{water} = d x V = 2.8 g/mL (c) $m_{\text{foam block}}$=2.76 g (same as mass of water displaced)
6. (a) 7.04 × 10^2; (b) 3.344 × 10^−2; (c) 5.479 × 10^2; (d) 2.2086 × 10^4; (e) 1.00000 × 10^3; (f) 6.51 × 10^−8; (g) 7.157 × 10^−3
7. (a) exact; (b) exact; (c) uncertain; (d) exact; (e) uncertain; (f) uncertain
8. (a) two; (b) three; (c) five; (d) four; (e) six; (f) two; (g) five
9. (a) 0.44; (b) 9.0; (c) 27; (d) 140; (e) 1.5 × 10^−3; (f) 0.44
10. (a) 2.15 × 10^5; (b) 4.2 × 10^6; (c) 2.08; (d) 0.19; (e) 27,440; (f) 43.0
11. (a) Archer X; (b) Archer W; (c) Archer Y

12. (a) 1.0936 yd m^{-1} ; (b) 0.94635 L qt^{-1} ; (c) 2.2046 lb kg^{-1}
13. 2.0 L (67.6 fl oz)-1=0.030 L (1 fl oz)-1
14. 68–71 cm; 400–450 g
15. 355 mL
16. 8×10^{-4} cm
17. yes; weight = 89.4 kg
18. 5.0×10^{-3} mL
19. (a) 1.3×10^{-4} kg; (b) 2.32×10^{8} kg; (c) 5.23×10^{-12} m; (d) 8.63×10^{-5} kg; (e) 3.76×10^{-1} m; (f) 5.4×10^{-5} m; (g) 1×10^{12} s; (h) 2.7×10^{-11} s; (i) 1.5×10^{-4} K
20. 45.4 L
21. 1.0160×10^{3} kg
22. (a) 394 ft; (b) 5.9634 km; (c) 6.0×10^{2}; (d) 2.64 L; (e) 5.1×10^{18} kg; (f) 14.5 kg; (g) 324 mg
23. 0.46 m; 1.5 ft/cubit
24. Yes, the acid's volume is 123 mL.
25. 62.6 in (about 5 ft 3 in.) and 101 lb.
26. (a) 3.81 cm × 8.89 cm × 2.44 m; (b) 40.6 cm
27. 2.70 g/cm^{3}
28. (a) 81.6 g; (b) 17.6 g
29. (a) 5.1 mL; (b) 37 L
30. 5371 °F, 3239 K
31. −23 °C, 250 K
32. −33.4 °C, 239.8 K
33. 113 °F