Fluid Dynamics

Read Chapter 12

<https://openstax.org/books/college-physics/pages/12-introduction-to-fluid-dynamics-and-its-biological-and-medical-applications>

**Formulas**

**Flow rate of a liquid**

$$Q= \frac{V}{t},$$

$Q$ is flow rate, SI units are m3/s;

$V$ is volume, SI units are m3; $t$ is time, SI units are s.

$Q=A\overbar{v}$,

$Q$ is flow rate, SI units are m3/s;

$A$ is cross-sectional area of the flow, SI units are m2,

$\overbar{v}$ is average velocity of the flow

For **incompressible fluids**, flow rate at various points is constant. That

$$Q\_{1}=Q\_{2}$$

$$A\_{1}\overbar{v\_{1}}=A\_{2}\overbar{v\_{2}}$$

$$n\_{1}A\_{1}\overbar{v\_{1}}=n\_{2}A\_{2}\overbar{v\_{2}}$$

$n\_{1}$ and $n\_{2}$ are the number of branches in each of the sections along the tube.

**Bernoulli’s equation**

$$P\_{1}+\frac{1}{2}ρv\_{1}^{2}+ρgh\_{1}=P\_{2}+\frac{1}{2}ρv\_{2}^{2}+ρgh\_{2}$$

$P$ is pressure, 𝞺 is density of a fluid, $h$ is height of a fluid, $g$ is acceleration due to gravity, $v$ is velocity of a fluid.

**Bernoulli’s principle**

h is constant, so $h\_{1}=h\_{2}$,

$$P\_{1}+\frac{1}{2}ρv\_{1}^{2}=P\_{2}+\frac{1}{2}ρv\_{2}^{2}$$

**Laminar flow** is characterized by smooth flow of the fluid in layers that do not mix.

**Turbulence** is characterized by eddies and swirls that mix layers of fluid together.

**Flow**

$Q= \frac{P\_{2}-P\_{1}}{R}$,

$Q$ is flow, $P$ is pressure, $R$ is resistance.

**Pressure drop**

$$P\_{1}-P\_{2}=QR$$

**Poiseuille’s law for flow in a tube**

$$Q=\frac{(P\_{2}-P\_{1})πr^{4}}{8ηl}$$

$Q$ is flow, $P$ is pressure, $r$ is radius of a tube, π is 3.14, 𝞰 is viscosity of a liquid, $l$ is length of a tube.

**Problems chapter 12**

[**https://openstax.org/books/college-physics/pages/12-problems-exercises**](https://openstax.org/books/college-physics/pages/12-problems-exercises)

**#2**

*The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to . (b) What is this rate in ?*

**Read again:**

*The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to . (b) What is this rate in ?*

**We know:** flow rate which is volume per unit time.

**We need to convert into different units.**

**Facts:**

1 L = 1000 cm3.

1 m3 = 1,000,000 cm3 = 106 cm3.

1 min = 60 seconds.

**Calculations**

**a)**

$\left(\frac{5.00 L}{1 min}\right)\left(\frac{1000 cm^{3}}{1 L}\right)\left(\frac{1 min}{60 s}\right)$= 83.3 cm3/s

**b)**

$\left(83.3\*\frac{1 m^{3}}{10^{6} cm^{3}}\right)=8.33\* 10^{-7}$ m3/s

**#4**

*Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.*

**Read again:**

*Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.*

**We know:** radius of a tube, velocity (rate in cm/s).

**We need to find:** flow rate and volume.

Since we have concepts such as flow rate, rate – average velocity, and volume; also, since we know the radius, we could find the area of a circle.

**Formula**

**Flow rate of a liquid**

$$Q= \frac{V}{t},$$

$Q$ is flow rate, SI units are m3/s;

$V$ is volume, SI units are m3; $t$ is time, SI units are s.

$Q=A\overbar{v}$,

$Q$ is flow rate, SI units are m3/s;

$A$ is cross-sectional area of the flow, SI units are m2,

$\overbar{v}$ is average velocity of the flow

**Area of a circle**

$A= πr^{2}$,

$A$ is area of a circle, $r$ is radius, π = 3.14.

**Calculations**



**Note:** for this type of questions, they report in cm3/s. SI units will be m3/s.

**# 6**

*A major artery with a cross-sectional area of  branches into 18 smaller arteries, each with an average cross-sectional area of . By what factor is the average velocity of the blood reduced when it passes into these branches?*

**Read again:**

*A major artery with a cross-sectional area of  branches into 18 smaller arteries, each with an average cross-sectional area of . By what factor is the average velocity of the blood reduced when it passes into these branches?*

**We know:** area of a larger tube (artery), area of a smaller tube (smaller arteries), number of smaller tubes (smaller arteries).

**We need to find:** factor by what the average velocity of a liquid is reduced

$$factor= \frac{\overbar{v\_{2}}}{\overbar{v\_{1}}}$$

Note: This factor does not have units.

**Formula**

We will use a formula for flow rate of incompressible liquid.

For **incompressible fluids**, flow rate at various points is constant. That

$$Q\_{1}=Q\_{2}$$

$$A\_{1}\overbar{v\_{1}}=A\_{2}\overbar{v\_{2}}$$

$$n\_{1}A\_{1}\overbar{v\_{1}}=n\_{2}A\_{2}\overbar{v\_{2}}$$

$n\_{1}$ and $n\_{2}$ are the number of branches in each of the sections along the tube.

Index 1 belongs to the major artery.

Index 2 belongs to smaller arteries.

In this problem, flow rate in a larger tube ($n\_{1}=1$) is the same as flow rate in all smaller tubes combined ($n\_{2}=18$).

**Calculations**



Now let’s find by what factor is the average velocity reduced:

$$factor= \frac{\overbar{v\_{2}}}{\overbar{v\_{1}}}=\frac{0.139\overbar{v\_{1}}}{\overbar{v\_{1}}}=0.139$$

This means that speed $\overbar{v\_{2}}$ equals 0.139 times speed $\overbar{v\_{1}}$, which is smaller than $\overbar{v\_{1}}$.

**# 7**

*(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is ?*

**Read again:**

*(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is ?*

**We know:** total area of larger tubes (venules), and factor by which the speed is increased, diameter of a small tube (a capillary).

**We need to find:** total area of smaller tubes (capillaries), number of smaller tubes (capillaries).

**Formula**

We will use a formula for flow rate of incompressible liquid.

For **incompressible fluids**, flow rate at various points is constant. That

$$Q\_{1}=Q\_{2}$$

$$A\_{1}\overbar{v\_{1}}=A\_{2}\overbar{v\_{2}}$$

$$n\_{1}A\_{1}\overbar{v\_{1}}=n\_{2}A\_{2}\overbar{v\_{2}}$$

$n\_{1}$ and $n\_{2}$ are the number of branches in each of the sections along the tube.

In this problem,

Index 1 belongs to capillaries, e.g.., $A\_{1}$ is the area of smaller tubes (capillaries),

Index 2 belongs to venules, e.g., $A\_{2}$ is the area of larger tubes (venules).

**Calculations**

1. We will use values for an area of venules and the factor by which the blood speed increased.

Note that the factor is the ratio of the average speed in capillaries to the average speed in venules:

 $factor= \frac{\overbar{v\_{2}}}{\overbar{v\_{1}}}$=4.00

**Calculations**



**b)**

To find the number of capillaries, we will divide the total area from a) by the area of one capillary. The area of one capillary can be found by using the formula for the area of a circle:

$A=πr^{2}$ , $r$ is radius and $r=\frac{d}{2}$, $d$ is diameter.

Note: the area in a) is in cm2, but the diameter of a capillary is in μm.

The prefix micro (μ) means 10-6.

Make sure you convert the diameter into cm as well before substituting into the formula.



**# 31**

*A glucose solution being administered with an IV has a flow rate of . What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.*

**Read again:**

*A glucose solution being administered with an IV has a flow rate of . What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.*

**We know:**

Flow rate of glucose, viscosity of blood is 2,50 times more that of the glucose, blood has the same density as glucose.

**We need to find:**

Flow rate of blood

**Formula**

**Poiseuille’s law for flow in a tube**

$$Q=\frac{(P\_{2}-P\_{1})πr^{4}}{8ηl}$$

$Q$ is flow, $P$ is pressure, $r$ is radius of a tube, π is 3.14, 𝞰 is viscosity of a liquid, $l$ is length of a tube.

All factors such as pressure difference, radius, length are the same for glucose and blood.

The only quantities that changes is viscosity 𝞰.

We know that $\frac{viscosity of blood}{voscosity of glucose}=2.50$ and the flow rate Q of glucose.

**Calculations**

We will use Poiseuille’s law twice and find the ratio of flow rate of blood to flow rate of glucose $\frac{Q^{,}}{Q}$, then rearrange for flow rate of blood.

All quantities that are the same cancel out (such as π, $l$, $r^{4}$, 8, $(P\_{2}-P\_{1})$).



**# 33**

*A small artery has a length of  and a radius of . If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is .)*

**Read again**

*A small artery has a length of  and a radius of . If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is .)*

**Note:** viscosity of blood changes with temperature.

**We know:**

The length and radius of an artery, pressure drop, and temperature.

Temperature allows to find the right value for the viscosity of blood.

**We need to find:**

Flow rate

**Note:** all quantities are in SI units.

**Formula**

**Poiseuille’s law for flow in a tube**

$$Q=\frac{(P\_{2}-P\_{1})πr^{4}}{8ηl}$$

$Q$ is flow, $P$ is pressure, $r$ is radius of a tube, π is 3.14, 𝞰 is viscosity of a liquid, $l$ is length of a tube.

**Calculations**



Note: because the value in SI units are very small, sometimes the flow is reported in mm3/s and not in m3/s.