Patterning and Algebra

Primary and Junior Grades

The Big Ideas for Patterning

- patterns help students recognize ***relationships*** within and between patterns

- ***growing and Shrinking patterns*** **involve an *increase or decrease* in elements as the pattern continues which can be described numerically**

- An ***understanding of patterns in numbers and operations*** contributes to the development of algebraic thinking

November 30, 2016

AN INVESTIGATION IN PATTERNING …

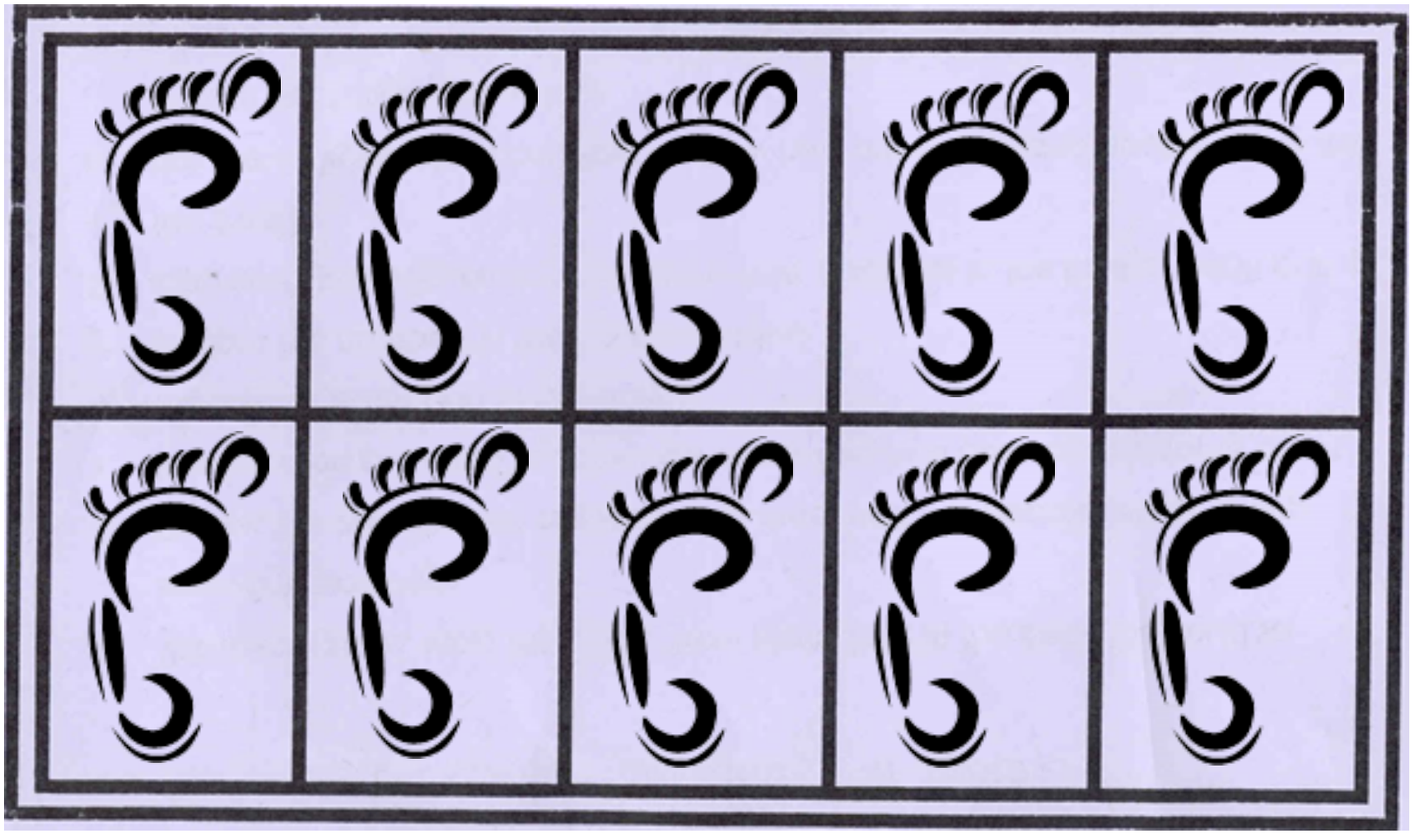
How Many Feet in the Bed?
String Cutting Task

Model & T-Chart

How Many Feet in the Bed?

Primary

10 Frame

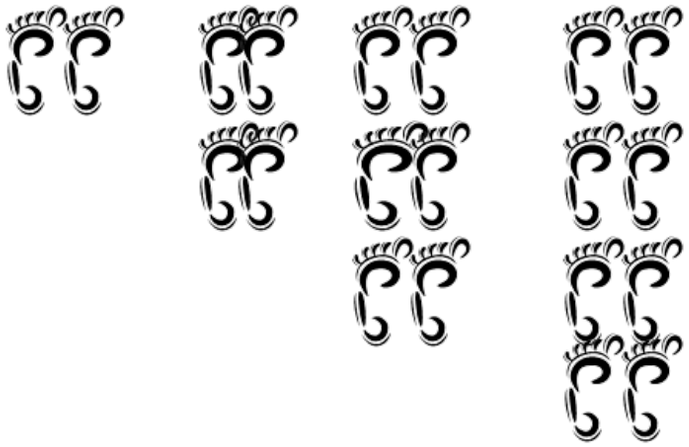


Displaying Data in a T Chart

Number of People	Number of Feet
1	2
2	
3	
4	

Modeling Math Meaningfully

Diagram or Manipulative



1 2 3 4

T-Chart

Number of People	Number of Feet
1	2
2	4
3	6
4	8

Rule

Graph

The Big Ideas in Algebra

- representing **mathematical relationships**
- **explaining relationships** between quantities
- **analyzing change**

Expressions & Equality
BIG IDEA 1:
Equations express the equality between quantities

- Understanding the = (equal) sign is critical:

3 + 3 = 2 + 4

5 = 5

5 = 3 + 2

3 + 2 = 4 + 1

- Watch your language … instead of:

"gives an answer of" or "make"

USE

"the same as"

- Need to recognize, define, create & maintain equality

Expressions & Equality

BIG IDEA 2: DOING & UNDOING

Seeing the ***general in the particular***

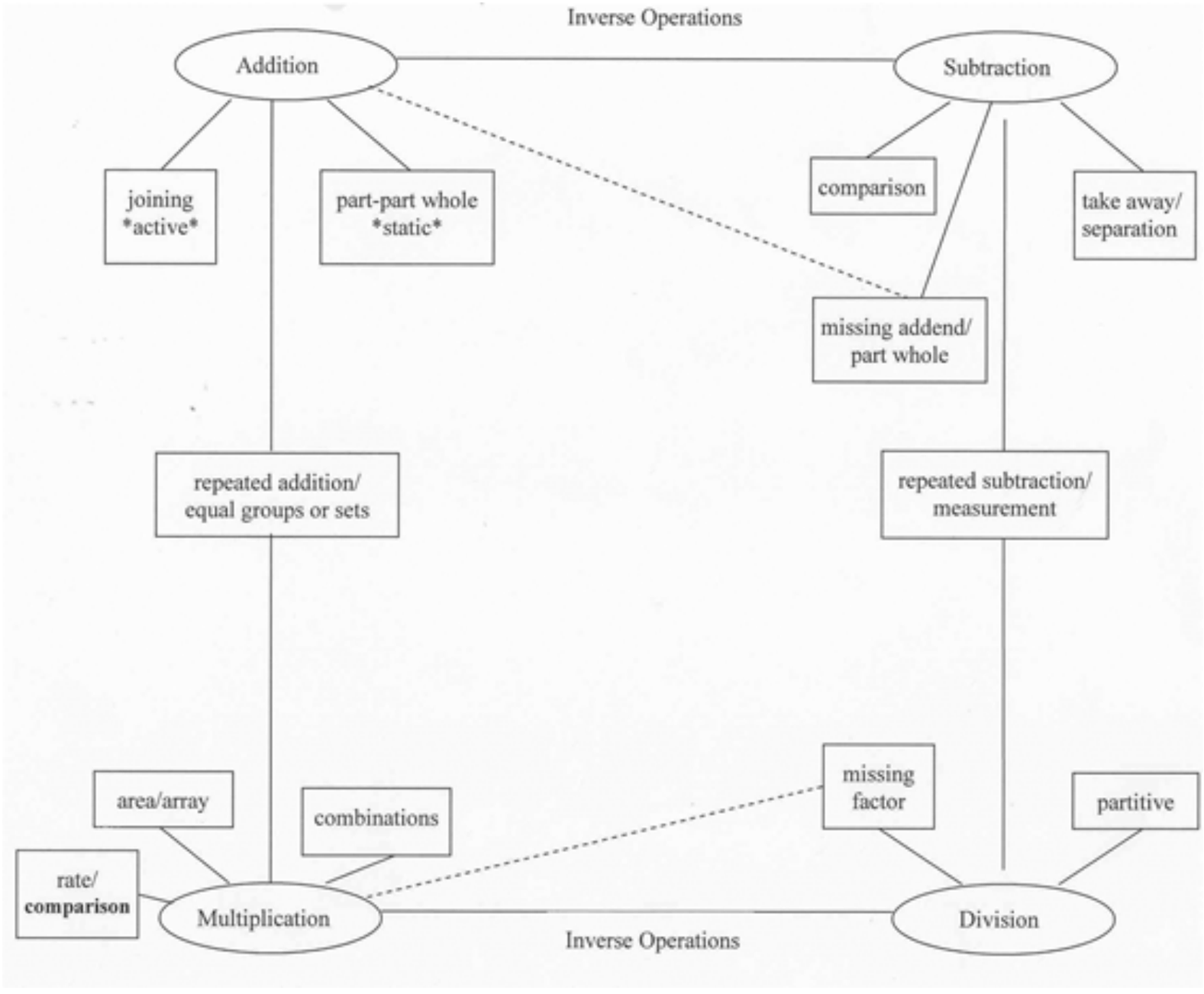
Need to understand the properties of numbers & operations as this helps develop the conceptual foundation for making sense of algebraic ideas

OAME, December 2008

Mathematical Properties

Property	Mathematical Language	Child's Language	How It Helps
Commutative	For all numbers a and b: a + b = b + a and a x b = b x a	If 4 + 7 = 11, then 7 + 4 must equal 11, too. If I know 4 x 7, I also know 7 x 4	The number of addition or multiplication facts to be memorized is reduced from 100 to 55.
Associative	For all numbers a, b, and c: (a + b) + c = a + (b + c) and (ab)c = a(bc)	When I'm adding (or multiplying) three or more numbers, it doesn't matter where I start.	When more than two numbers are being added (or multiplied), combinations that make the task easier can be chosen. For example, 37 x 5 x 2 can be done as 37 x (5 x 2) or 37 x 10 Rather than (37 x 5) x 2.
Distributive	For all numbers a, b, and c: a(b + c) = ab + ac	(6 x 15) is the same as (6 x 10) + (6 x 5) 96 ÷ 3 is the same as (90 ÷ 3) + (6 ÷ 3)	Some of the more difficult basic facts can be split into smaller, easier-to-remember parts. For example, 6 x 15 is the same as (6 x 10) + (6 x 5) or 60 + 30 The 19 addition facts involving 0 and the 19 multiplication facts involving 1 can be easily remembered once this property is understood and established.
Role of Zero in (x)	For any inside number a: 0 x a = 0 or a x 0 = 0	0 multiplied by any number is 0. Any number multiplied by 0 is 0	The 19 multiplication facts involving zero can be generalized.

Other Arithmetic Generalizations



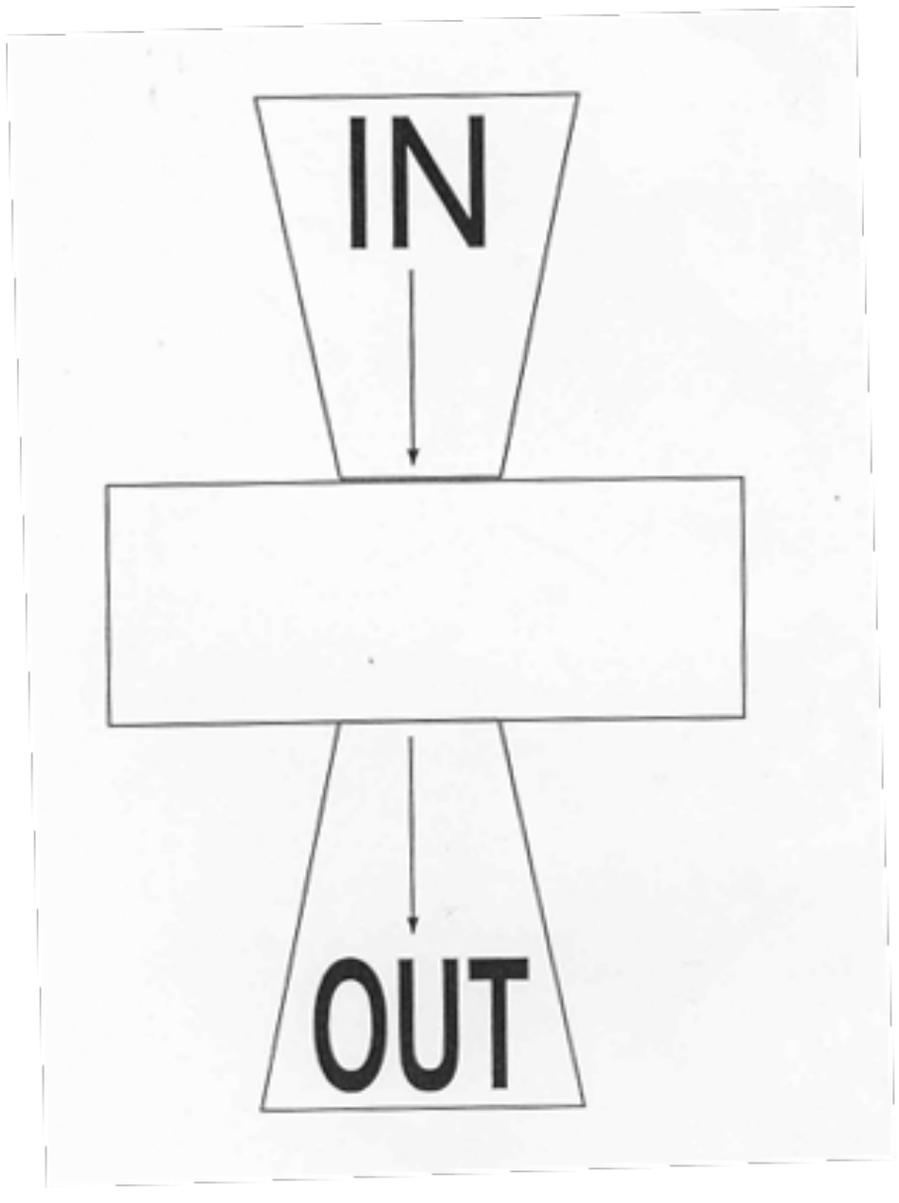
Variables

- **Variables are used to represent unknown quantities: (□ + 4 = 12), to represent quantities that vary: (4 +▲ = □), and to generalize number properties: (a + b = b + a)**

- **Formulas are algebraic equations showing a relationship between two or more different quantities: (V = l x w x h)**

Algebra as Functional Relationships

Function Machines



In a marble machine, the number of marbles inputted and outputted were recorded. One boy put in 1 marble and got back 3 marbles. One girl put in 9 marbles and got back 11 marbles and so on. What is the rule?

Input	Output
1	3
9	11
2	4
6	8
30	32
3	5
21	23

OAME, March 2009

THE INVESTIGATION CONTINUES ...

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&
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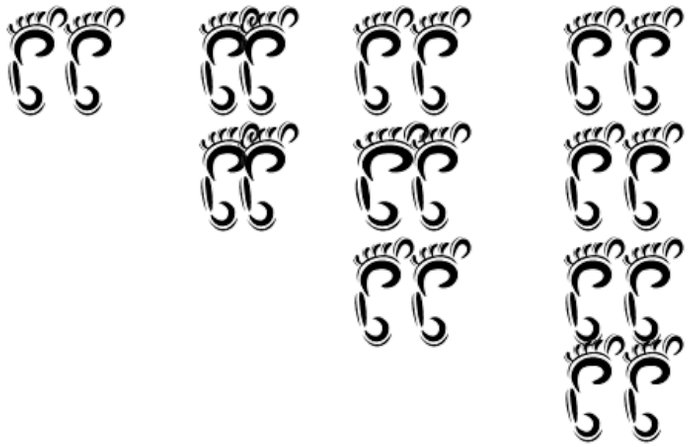
Modeling Math Meaningfully

Diagram or Manipulative



1 2 3 4

T-Chart

Input ☐ Output ▲

Number of People	Number of Feet
1	2
2	4
3	6
4	8

Rule

Graph

Finding Correspondence Between Quantities

Number of People	Number of Feet
1	2
2	4
3	6
4	8

quadruple

x 4

?

Algebra – Expressing Generality Using Multiple Representations

BIG IDEA:

"Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations." (p.18)

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. Journal of Mathematics Education Leadership, (7)3, 9-24.

Completing the Making Math Meaningful Chart …

Suppose you wanted to find the number of shapes needed in the "nth" term?

1)Determine the generalized rule for the pattern.

2)Draw a graph to represent the growing pattern.

THE INVESTIGATION CONTINUES ...

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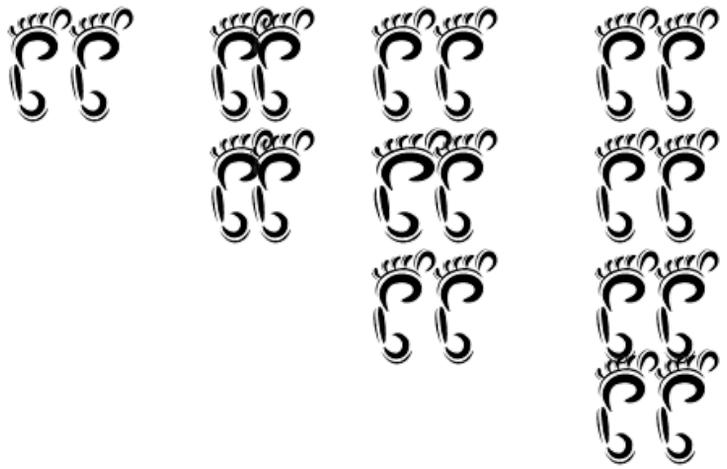


1 2 3 4

T-Chart

Input ☐ Output ▲

Number of People	Number of Feet
1	2
2	4
3	6
4	8

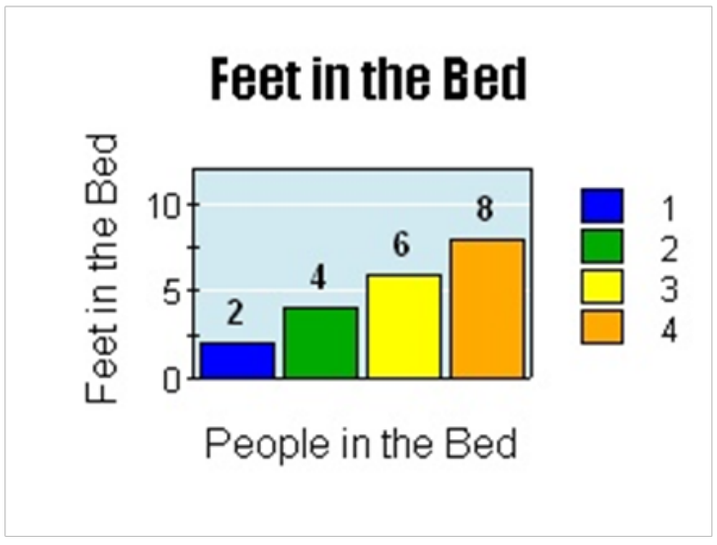
Suppose you wanted to find out the total number of feet for any number of people?

Generalized or Algebraic Equation:

2 x number of people = number of feet in the bed

2 x ☐ = ▲

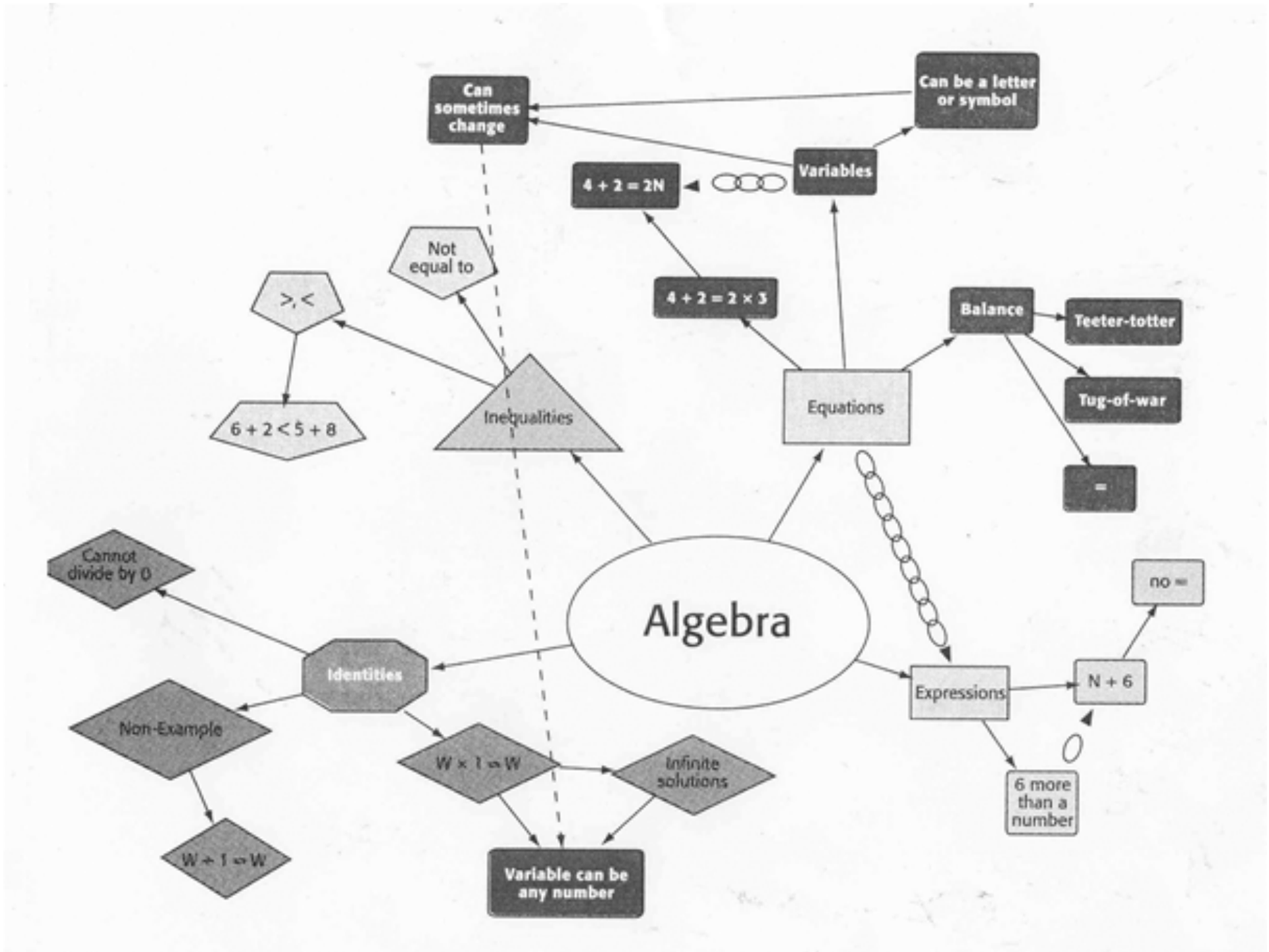
Rule



Graph

To help **JUNIOR** students develop an understanding of Patterning and Algebra we can help them to:

- ***Investigate*** patterning and algebra problems in real life settings, and learn to calculate any term in a pattern when given the term number
- ***Extend*** their knowledge of generating patterns that involve +,-,x, division as well as those involving reflections, translations or rotations
- ***Investigate*** problems involving missing numbers and develop an early sense of variables
- ***Extend*** their understanding of equality of expressions using multiplication or division in equations with unknown quantities on both side


Algebra
Can sometimes change
Can be a letter or symbol
Variables
4 + 2 = 2N
Not equal to
>, <
6 + 2 < 5 + 8
Inequalities
4 + 2 = 2 × 3
Equations
Balance
Teeter-totter
Tug-of-war
=
no =
Cannot divide by 0
Identities
Non-Example
W ÷ 1 ⇔ W
W × 1 ⇔ W
Infinite solutions
Variable can be any number
Expressions
N + 6
6 more than a number
0