Fundamentals of Business Math Exercises

A companion to [Fundamentals of Business Math](#)

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Attributions

This collection is a work in progress. If you notice errors or omissions, please contact the principal author at ana.duff@ontariotechu.ca.

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Contents

Chapter 1 Review of the Basics

1.1 Algebraic Expressions

Mechanics

For the following four questions, simplify the algebraic expressions.

1) $2a - 3a + 4 + 6a - 3$

2) $5b(4b + 2)$

3) $\frac{6x^3 + 12x^2 + 13.5x}{3x}$

4) $(1 + i)^3 \cdot (1 + i)^{14}$

5) Evaluate the power: $8^{2/3}$

6) Substitute the known variables and solve for the unknown variable:

$$I = Prt \text{ where } P = \$2{,}500, r = 0.06, \text{ and } t = \frac{135}{365}$$

Applications

For the following five questions, simplify the algebraic expressions.

7) $(6r^2 + 10 - 6r + 4r^2 - 3) - (-12r - 5r^2 + 2 + 3r)$

8) $\frac{5x^9 + 3x^9}{2x}$

9) $\frac{t}{2} + 0.75t - t^3 + \frac{5t^4}{t} - \frac{2(t + t^3)}{4}$

10) $\frac{14(1+i) + 21(1+i)^4 - 35(1+i)^7}{7(1+i)}$

11) $\frac{R}{1 + 0.08} \cdot \frac{183}{365} + 3R(1 + 0.08) \cdot \frac{52}{365}$

12) Evaluate the power: $\left[\left(\frac{2}{5} \right)^2 \right]^2$

For questions 13 and 14, substitute the known variables and solve for the unknown variable.

13) $PV = \frac{FV}{(1+i)^N}$ where $FV = \$3{,}417.24, i = 0.05,$ and $N = 6$.

14) $PMT = \frac{FV}{\left[\frac{(1+i)^N - 1}{i} \right]}$ where $FV = \$10{,}000, N = 17,$ and $i = 0.10$.

Challenge, Critical Thinking, and Other Applications

15) Simplify

$$\left[\frac{10a^2b^3x^4}{5b^3c^4}\right]^2 + 6(a^8)^{\frac{1}{2}} - (3a^2+6)(3a^2-3)$$

16) Simplify

$$\frac{-(5x+4y+3)(2x-5y) - (10x-2y)(2y+3)}{5}$$

17) Simplify

$$\frac{(-3z)^3}{(3z^2)^2}(2z^3)^{-4}$$

18) Substitute the given variable values and solve for the unknown in:

$$FV_{ORD} = PMT(1+\Delta)^{N-1}\left[\frac{\left[\frac{(1+i)^{\frac{CY}{PY}}}{(1+\Delta)}\right]^N - 1}{\frac{(1+i)^{\frac{CY}{PY}}}{(1+\Delta)} - 1}\right]$$

where $PMT = 500, i = 0.05, \Delta = 0.02, CY = 2, PY = 4,$ and $N = 20$.

19) Evaluate

$$50{,}000 \cdot \left(1 + \frac{0.10}{12}\right)^{-27}$$

20) Evaluate

$$995\left[\frac{1 - (1+0.02)^{13}\left(1+\frac{0.09}{4}\right)^{-13}}{\frac{0.09}{4} - 0.02}\right]$$

1.2 Linear Equations - Manipulating and Solving

Mechanics

Solve the following equations for the unknown variable.

21) $3(x-5) = 15$

22) $12b - 3 = 4 + 5b$

23) $0.75(4m+12) + 15 - 3(2m+6) = 5(-3m+1) + 25$

Solve each of the following pairs of equations for both unknown variables.

24) $x + y = 6; \quad 3x - 27 = 8$

25) $4h - 7q = 13; \quad 6h + 3q = 33$

26) $0.25a + \frac{5b}{2} = 3.5; \quad \frac{3a}{4} - \frac{b}{0.2} = 3$

Applications

In the following two questions, solve the equation for the unknown variable.

27) $\frac{4y}{1.025^4} + y - 2y(1.05)^2 = 1{,}500$

28) $2{,}500(1 + 0.06t) + 1{,}000(1 + 0.04t) = 3{,}553.62$

29) Pamela is cooking a roast for a 5:30 p.m. dinner tonight. She needs to set a delay timer on her oven. The roast takes 1 hour and 40 minutes to cook. The time right now is 2:20 p.m. How long of a delay must she set the oven for (before it automatically turns on and starts to cook the roast)?

30) In 2010, 266 million North Americans were using the Internet, which represented a 146.3% increase in Internet users over the year 2000. How many North American Internet users were there in 2000?

31) A human resource manager is trying to estimate the cost of a workforce accident. These costs usually consist of direct costs (such as medical bills, equipment damage, and legal expenses) and indirect costs (such as decreased output, production delays, and fines). From past experience, she knows that indirect costs average six times as much as direct costs. If she estimates the cost of an accident to be $21,000, what is the direct cost of the accident?

32) In 2011, Canadian federal tax rates were 0% on the first $10,527 of gross income earned, 15% on the next $31,017, 22% on the next $41,544, 26% on the next $45,712, and 29% on anything more. If a taxpayer paid $28,925.35 in federal tax, what was her gross annual income for 2011?

33) St. Boniface Hospital raises funds for research through its Mega Lottery program. In this program, 16,000 tickets are available for purchase at a price of one for $100 or three for $250. This year, the lottery sold out with sales of $1,506,050. To better plan next year's lottery, the marketing manager wants to know how many tickets were purchased under each option this year.

34) An accountant is trying to allocate production costs from two different products to their appropriate ledgers. Unfortunately, the production log sheet for last week has gone missing. However, from other documents he was able to figure out that 1,250 units in total were produced last week. The production machinery was run for 2,562.5 minutes, and he knows that Product A takes 1.5 minutes to manufacture while Product B takes 2.75 minutes to manufacture. How many units of each product were produced last week?

Challenge, Critical Thinking, and Other Applications

35) Jacob owns 15,000 shares in a corporation, which represents 2% of all issued shares by the corporation. He sold off his shares to another investor for $7,800. What is the total value for all of the shares issued by the corporation?

36) Two cellphone companies are offering different rate plans. Rogers is offering $19.99 per month, which includes a maximum of 200 weekday minutes plus $0.35 for every minute above the maximum. TELUS is offering $39.99 for a maximum 300 weekday minutes, but it charges $0.10 for every minute above the maximum. Above how many minutes would TELUS be the better choice?

37) Marianne, William, Hendrick, and Charlotte have all decided to go into business together. They need $175,000 in initial capital funding. William was able to contribute

20% less than Marianne, Hendrick contributed 62.5% more than William, and Charlotte contributed $5,000 less than half as much as Marianne. How much did each partner contribute to the initial funds?

38) A mall is being constructed and needs to meet the legal requirements for parking availability. Parking laws require one parking stall for every 100 square feet of retail space. The mall is designed to have 1,200,000 square feet of retail space. Of the total parking stalls available, 2% need to be handicap accessible, there need to be three times as many small car spaces as handicap spaces, RV spaces need to be one-quarter of the number of small car spaces, and the rest of the spaces are for regular stalls. How many of each type of parking space does the mall require?

39) Rearrange the following equation to solve for z and find the solution. Verify the solution through substitution.

$$z\left(1 + 0.073 \cdot \frac{280}{365}\right) - \frac{z}{1 + 0.073 \cdot \frac{74}{365}} + 1000 = 2{,}764.60$$

40) Find the solutions for the following pair of equations. Verify the solution through substitution into both equations.

$$3\frac{4}{5}q + 0.18r = 12.2398$$
$$-5.13q - \frac{133r}{5} = -38.4674$$

1.3 Writing Expressions and Solving Equations in a General Setting

Re-arrange each of the following equations to solve for the given variable.

41) $N = rA$; for A

42) $P_1 = P_0 + rP_0$; for r

43) $px = Vx + F$; for x

44) $T = n(t + h + rt + ht)$; for t

45) $k = \frac{D_1}{P_0} + g$; for D_1

46) $P_0 = \frac{D_0(1+g)}{k-g}$; for g

47) $s_p = ws_A - (1-w)s_B$; for w

48) $k = \frac{D_1}{P_0} + g$; for g

49) $N = L(1 - d_1)(1 - d_2)$; for d_1

50) $P_0 = \frac{D_0(1+g)}{k-g}$; for k

51) $C = \frac{1}{h}\left(S - \frac{P}{1+r}\right)$; for P

52) $k = \frac{D_1}{P_0} + g$; for P_0

53) $\frac{I_1}{I_2} = \frac{S_1}{S_2}$; for I_2

54) $N = P + rP$; for P

55) $px = Vx + F$; for V

56) $P_1 = P_0 + rP_0$; for P_0

57) $h = \frac{PU - PD}{PU - X}$; for PU

58) $C = \frac{1}{h}\left(S - \frac{P}{1+r}\right)$; for r

59) $F = (1 - c)S$; for c

60) $S = C + m_S S$; for S

61) $k - RF = (ER - RF)B$; for RF

62) $ER_p = RF + \left[\frac{ER_M - RF}{S_M}\right]S_P$; for S_p

63) $S = C + m_C C$; for m_c

64) $S = L(1 - d)$; for L

65) $C(1 + r_1) = \frac{S}{1 - r_2}$; for C

66) $\frac{I_1}{I_2} = \frac{S_1}{S_2}$; for S_1

Write the algebraic expressions that represent solutions to the problems below. Use the variables provided. Make sure that you use consistent capitalization of letters (i.e., f is not the same as F, and pf is not the same as p_f, for example).

67) Norachai sold F French silk cheesecakes at the price of p_F dollars per cake and M chocolate marble cheesecakes at p_M dollars per cake. Write the expression that describes the proceeds from her sale.

68) In Ontario, businesses are required to collect the Harmonized Sales Tax of 13% with each sale of a taxable good or service. If a retailer has made sales in the amount of S dollars, what is the total amount they collected from their customers?

69) Ticketmaster charges T dollars per ticket and a handling charge of H dollars for each ticket purchased. In addition, it charges an administrative fee F with each purchase. If the goods and services tax of 7% is applied on the ticket price, the handling charge and the administrative fee, what is the total amount that will be charged for a purchase of n tickets?

70) An OntarioTech student is looking at working over the summer as an independent dog walker for a company that would provide him with clients. The company charges the clients a pre-set amount of d dollars per dog per day. The student receives 75% of this amount but estimates it will cost them, on average, $1.50 per dog per day on pick-up and drop-off expenses, in addition to the daily parking charge of P dollars at the local dog park. How much profit will they make per dog if they walk n dogs per day?

71) You want to invest a sum P in three types of investments, equally distributed, at the interest rates of r_1, r_2 and r_3 per year respectively, for n years. How much will you have in total at the end of this period?

72) You are working in a sales job and are paid by commission at the rate of r_1 for the first A_1 dollars in sales, at the rate of r_2 for the next A_2 dollars in sales, and at the rate of r_3 for sales above that. If you worked h hours in the week in which you made S dollars in sales, how much did you earn, on average, per hour?

73) Marketing analysts for a media company have determined that f% of Facebook users will click on a funny ad on their Facebook wall and that i% of Instagram users will click on a funny ad on their Instagram wall. If the company pushes a funny ad to F Facebook users and I Instagram users, what percentage of the total number of users exposed to the ad can be expected to click on the ad on either of the social platforms?

74) You want to invest a sum P in three types of investments, A, B and C, at the interest rates of r_1, r_2 and r_3 per year respectively, for n years. The amount invested in A should be a quarter of the total amount, and the rest should be equally distributed between the other two investments. How much will you have in total at the end of this period?

75) S&A Electronics sold a stereo during a sale at some sale price. What is the regular selling price of the stereo if its price was reduced for the sale by one third of the regular selling price?

76) Some CDs were put on sale at 40% off. What was the regular price given the sale price?

77) This month's commodity index decreased by one twelfth of last month's index. If you are given this month's index, how much is the last month's index?

78) Jay purchased tickets for a concert over the Internet. To place the order, a handling charge was charged on each ticket. HST of 13% was also charged on the ticket price and the handling charge. If you know the total charge Jay paid for the tickets and the number of tickets he purchased, what was the total cost per ticket before tax?

79) Ken and Martina agreed to form a partnership. The partnership agreement requires Martina invest $2500 more than two thirds of what Ken is to invest. If you know the total partnership's capital, how much must Martina invest?

80) Giuseppi's Pizza filled an order for the FBIT Student Association's welcome party. If the number of meat lovers pizzas was one less than three times the number of pepperoni pizzas, and the number of vegetarian pizzas was one more than twice the number of peperoni pizzas, how many pizzas of each type were ordered if you are given the total amount owed for the order and the price of each pizza?

1.4 Effect of change in multivariable relationships

81) Analyze how a change (increase or decrease) in each of the input quantities affects the change in the output quantity in the following relationships related to finance and sketch a graph representing this relationship. Assume all variables represent positive values.

a) Future value: $FV_n = C_0 \cdot (1 + r)^n$

b) Present value: $PV_0 = \frac{C_n}{(1+r)^n}$

c) Present value of a stream of cash flows:

$$PV_0 = C_0 + \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} + \cdots + \frac{C_n}{(1 + r)^n}$$

d) Present value of growing perpetuity: $PV_0 = \frac{C_1}{r-g}$

e) Constant Dividend Growth Model: $P_0 = \frac{Div_1}{r_E - g}$

f) $real\ interest\ rate = \frac{nominal\ rate - inflation\ rate}{1 + inflation\ rate}$

g) $net\ working\ capital = current\ assets - current\ liabilities$

h) $current\ liquidity\ ratio = \frac{total\ current\ assets}{total\ current\ liabilities}$

i) $total\ asset\ turnover = \frac{total\ operating\ revenues}{average\ total\ assets}$

j) $earnings\ before\ interest\ and\ taxes = (interest\ coverage)(interest\ expense)$

k) $debt{-}equity\ ratio = \frac{total\ debt}{total\ equity}$

l) $return\ on\ assets = (profit\ margin)(asset\ turnover)$

m) $external\ funds\ needed =$
$\left(\frac{assets - debt}{sales}\right)(projected\ change\ in\ sales)$
$-(net\ profit\ margin)(projected\ sales)(1 - dividend\ payout\ ratio)$

82) Analyze how the change in each of the input quantities affects the change in the output quantity in the following relationships related to accounting and sketch a graph representing this relationship. Assume all variables represent positive values.

a) $current\ ratio = \frac{current\ assets}{current\ liabilities}$

b) $quick\ ratio = \frac{current\ assets - inventory - prepaid\ expenses}{current\ liabilities}$

c) $average\ collection\ period = \frac{365\ days}{accounts\ receivable\ turnover}$

d) $net\ debt\ as\ a\ percentage\ of\ total\ capitalization = \frac{net\ debt}{shareholders'equity + net\ debt}$

e) $return\ on\ equity = \frac{net\ income - preferred\ dividends}{average\ common\ shareholders'equity}$

83) Analyze how the change in each of the input quantities affects the change in the output quantity in the following relationships related to operations management and sketch a graph representing this relationship. Assume all variables represent positive values.

a) $expected\ output = (effective\ capacity) \cdot (efficiency)$

b) $cycle\ time = \frac{production\ time\ available\ per\ day}{units\ required\ per\ day}$

c) $standard\ time = \frac{total\ normal\ time}{1 - allowance\ factor}$

d) $crash\ cost\ per\ period = \frac{crash\ cost - normal\ cost}{normal\ time - crash\ time}$

e) $break\text{-}even\ in\ units = \frac{total\ fixed\ cost}{price - variable\ cost}$

f) $break\text{-}even\ in\ dollars = \frac{total\ fixed\ cost}{1 - \frac{variable\ cost}{price}}$

84) Analyze how the change in each of the input quantities affects the change in the output quantity in the following relationships related to marketing and sketch a graph representing this relationship. Assume all variables represent positive values.

a) $increase\ in\ sales = \frac{increase\ in\ fixed\ costs}{contribution\ margin}$

b) $sales\ force\ size = \frac{(\#\ of\ salespeople) \cdot (\#\ of\ customers) \cdot (avg.\ length\ of\ customer\ call)}{time\ an\ average\ salesperson\ has\ available\ for\ selling\ per\ year}$

c) $unit\ volume = \frac{fixed\ cost - profit\ goal}{price - variable\ cost}$

d) $sales = \frac{fixed\ cost + profit\ goal}{contribution\ margin}$

e) $unit\ cost = variable\ cost + \frac{fixed\ costs}{unit\ sales}$

f) $markup\ price = \frac{unit\ cost}{1 - desired\ return\ on\ sales}$

g) $ROI\ price = unit\ cost + \frac{ROI \cdot investment}{unit\ sales}$

h) $dollar\ markup = selling\ price - cost$

i) $markup\ percentage\ on\ cost = \frac{dollar\ markup}{cost}$

j) $markup\ percentage\ on\ selling\ price = \frac{dollar\ markup}{selling\ price}$

85) Analyze how the change in each of the input quantities affects the change in the output quantity in the following relationships related to human resources management and sketch a graph representing this relationship. Assume all variables represent positive values.

a) $turnover\ rate = \frac{number\ of\ employees\ who\ leave}{total\ number\ of\ employees}$

b) $absence\ rate = \frac{\#\ days\ absent\ in\ month}{(avg.\#of\ employees\ during\ a\ month)\cdot(\#\ of\ workdays)}$

c) $benefits\ as\ a\ percent\ of\ salary = \frac{annual\ benefits\ costs}{annual\ salary}$

d) $cost\ per\ hire = \frac{recruitment\ costs}{compensation\ cost+benefit\ cost}$

e) $turnover\ costs = separation + vacancy + replacement + training$

Chapter 2 General Business Management Applications

2.1 Percent Change

Mechanics

For the following three questions, solve for the unknown (?) using the percent change formula.

	Old	New	Δ%
86)	$109.95	$115.45	?
87)	?	$622.03	13.25%
88)	5.94%	?	−10%

89) If $9.99 is changed to $10.49, what is the percent change?

90) $19.99 lowered by 10% is what dollar amount?

91) What amount when increased by 40% is $3,500?

92) If 10,000 grows to 20,000 over a period of 10 years, what is the annual rate of change?

Applications

93) How much, including taxes of 12%, would you pay for an item with a retail price of $194.95?

94) From September 8, 2007 to November 7, 2007, the Canadian dollar experienced a rapid appreciation against the US dollar, going from $0.9482 to $1.1024. What was the percent increase in the Canadian dollar?

95) From 1996 to 2006, the "big three" automakers in North America (General Motors, Ford, and Chrysler) saw their market share drop from 71.5% to 52.7%. What is the overall percent change and the rate of change per year?

96) The average price of homes in Calgary fell by $10,000 to $357,000 from June 2009 to July 2009. The June 2009 price was 49% higher than the June 2005 price.

   a) What was the percent change from June 2009 to July 2009?

   b) What was the average price of a home in June 2005?

   c) What was the annual rate of change from June 2005 to June 2009?

97) On October 28, 2006, Saskatchewan lowered its provincial sales tax (PST) from 7% to 5%. What percent reduction does this represent?

98) A local Superstore sold 21,983 cases of its President's Choice cola at $2.50 per case. In the following year, it sold 19,877 cases at $2.75 per case.

   a) What is the percent change in price year-over-year?

   b) What is the percent change in quantity year-over-year?

   c) What is the percent change in total revenue year-over-year?
   (Hint: $revenue = price \cdot quantity$)

99) A bottle of liquid laundry detergent priced at $16.99 for a 52-load bottle has been changed to $16.49 for a 48-load bottle. By what percentage has the price per load changed?

Challenge, Critical Thinking, and Other Applications

100) At a boardroom meeting, the sales manager is happy to announce that sales have risen from $850,000 to $1,750,000 at a rate of 4.931998% per year. How many years did it take for the sales to reach $1,750,000?

101) The Nova Scotia Pension Agency needs to determine the annual cost of living adjustment (COLA) for the pension payments made to its members. To do this, it averages the consumer price index (CPI) for both the previous fiscal year and the current fiscal year. It then calculates the percent change between the two years to arrive at the COLA. If CPI information is as follows, determine the COLA that the pensioners will receive.

Previous Fiscal Year		Current Fiscal Year	
Nov. 109.2	May 112.1	Nov. 111.9	May 114.6
Dec. 109.4	June 111.9	Dec. 112.0	June 115.4
Jan. 109.4	July 112.0	Jan. 111.8	July 115.8
Feb. 110.2	Aug. 111.7	Feb. 112.2	Aug. 115.6
Mar. 111.1	Sep. 111.9	Mar. 112.6	Sep. 115.7
Apr. 111.6	Oct. 111.6	Apr. 113.5	Oct. 114.5

102) During The Bay's warehouse clearance days, it has reduced merchandise by 60%. As a bonus, today is Scratch 'n' Save day, where you can receive up to an additional 25% off the reduced price. If you scratched the maximum of 25% off, how many dollars would you save off an item that is regularly priced at $275.97? What percent savings does this represent?

103) Federal Canadian tax rates for 2010 and 2011 are listed below. For example, you pay no tax on income within the first bracket, 15% on income within the next bracket, and so on. If you earned $130,000 in each year, by what percentage did your overall federal tax rate change? In dollars, what was the difference in tax?

2010 Tax Brackets	Taxed at	2011 Tax Brackets
$0 – $10,382	0%	$0 – $10,527
$10,383 – $40,970	15%	$10,528 – $41,544
$40,971 – $81,941	22%	$41,545 – $83,088
$81,942 – $127,021	26%	$83,089 – $128,800
$127,022+	29%	$128,801+

104) Melina is evaluating two colour laser printers for her small business. A Brother model is capable of printing 21 colour pages per minute and operates 162.5% faster than a similar Hewlett-Packard model. She needs to print 15,000 pages for a promotion. How much less time (stated as a percentage) will it take on the Brother model?

105) A chocolate bar has been priced at $1.25 for a 52 gram bar. Due to vending machine restrictions, the manufacturer needs to keep the price the same. To adjust for rising costs, it lowers the weight of the bar to 48 grams.

   a) By what percentage has the price per gram changed?

   b) If this plan is implemented over two periods, what rate of change occurs in each period?

2.2 Averages

Mechanics

In the following two questions, calculate the simple average.

106) Data   8   17   6   33   15   12   13   16

107) Data $1,500 $2,000 $1,750 $1,435 $2,210

In the following two questions, calculate a weighted average.

108) Data   4   4   4   4   12 12 12 12 12 12 12 15 15

109)

Data	$3,600	$3,300	$3,800	$2,800	$5,800
Weight	2	5	3	6	4

In the following two questions, calculate a geometric average. Round all percentages to four decimals.

110) Data   5.4%   8.7%   6.3%

111) Data   10%   -4%   17%   -10%

Applications

112) If a 298 mL can of soup costs $2.39, what is the average price per millilitre?

113) Kerry participated in a fundraiser for the Children's Wish Foundation yesterday. She sold 115 pins for $3 each, 214 ribbons for $4 each, 85 coffee mugs $7 each, and 347 baseball hats for $9 each. Calculate the average amount Kerry raised per item.

114) Stephanie's mutual funds have had yearly changes of 9.63%, -2.45%, and 8.5%. Calculate the annual average change in her investment.

115) In determining the hourly wages of its employees, a company uses a weighted system that factors in local, regional, and national competitor wages. Local wages are considered most important and have been assigned a weight of 5. Regional and national wages are not as important and have been assigned weights of 3 and 2, respectively. If the hourly wages for local, regional, and national competitors are $16.35, $15.85, and $14.75, what hourly wage does the company pay?

116) Canadian Tire is having an end-of-season sale on barbecues, and only four floor models remain, priced at $299.97, $345.49, $188.88, and $424.97. What is the average price for the barbecues?

117) Calculate the grade point average (GPA) for the following student. Round your answer to two decimals.

Course	Grade	Credit Hours		Grade	Grade Point
Economics 100	D	5		A+	4.5
Math 100	B	3		A	4.0
Marketing 100	C	4		B+	3.5
Communications 100	A	2		B	3.0
Computing 100	A+	3		C+	2.5
Accounting 100	B+	4		C	2.0
				D	1.0
				F	0.0

118) An accountant needs to report the annual average age (the length of time) of accounts receivable (AR) for her corporation. This requires averaging the monthly AR averages, which are listed below. Calculate the annual AR average.

Month	Monthly AR Average	Month	Monthly AR Average	Month	Monthly AR Average
January	$45,000	May	$145,000	September	$185,000
February	$70,000	June	$180,000	October	$93,000
March	$85,000	July	$260,000	November	$60,000
April	$97,000	August	$230,000	December	$50,000

119) From January 2007 to January 2011, the annual rate of inflation has been 2.194%, 1.073%, 1.858%, and 2.346%. Calculate the average rate of inflation during this period.

Challenge, Critical Thinking, and Other Applications

120) Gabrielle is famous for her trail mix recipe. By weight, the recipe calls for 50% pretzels, 30% Cheerios, and 20% peanuts. She wants to make a 2 kg container of her mix. If pretzels cost $9.99/kg, Cheerios cost $6.99/kg, and peanuts cost $4.95/kg, what is the average cost per 100 g rounded to four decimals?

121) Caruso is the marketing manager for a local John Deere franchise. He needs to compare his average farm equipment sales against his local Case IH competitor's sales. In the past three months, his franchise has sold six $375,000 combines, eighteen $210,000 tractors, and fifteen $120,000 air seeders. His sales force estimates that the Case IH dealer has sold four $320,000 combines, twenty-four $225,000 tractors, and eleven $98,000 air seeders. Express the Case IH dealer's average sales as a percentage of the John Deere dealer's average sales.

122) You are shopping for shampoo and consider two brands. Pert is sold in a bundle package of two 940 mL bottles plus a bonus bottle of 400 mL for $13.49. Head & Shoulders is sold in a bulk package of three 470 mL bottles plus a bonus bottle of 280 mL for $11.29.

a) Which package offers the best value?

b) If Head & Shoulders increases its package size to match Pert's package size while maintaining the same price per mL, how much money do you save by choosing the lowest priced package?

123) The following are annual net profits (in millions of dollars) over the past four years for three divisions of Randy's Wholesale:

Cosmetics: $4.5, $5.5, $5.65, $5.9
Pharmaceutical: $15.4, $17.6, $18.5, $19.9
Grocery: $7.8, $6.7, $9.87, $10.75

Rank the three divisions from best performing to worst performing based on average annual percent change.

124) You are shopping for a Nintendo Wii gaming console and visit www.shop.com, which finds online sellers and lists their prices for comparison. Based on the following list, what is the average price for a gaming console (rounded to two decimals)?

NothingButSoftware.com	$274.99
eComElectronics	$241.79
NextDayPC	$241.00
Ecost.com	$249.99
Amazon	$169.99
eBay	$165.00
Buy.com	$199.99
HSN	$299.95
Gizmos for Life	$252.90
Toys 'R' Us	$169.99
Best Buy	$169.99
The Bay	$172.69
Walmart	$169.00

125) Juanita receives her investment statement from her financial adviser at Great-West Life. Based on the information below, what is Juanita's average rate of return on her investments?

Investment Fund	Proportion of Entire Portfolio Invested in Fund	Fund Rate of Return
Real Estate	0.176	8.5%
Equity Index	0.073	36.2%
Mid Cap Canada	0.100	−1.5%
Candian Equity	0.169	8.3%
US Equity	0.099	−4.7%
US Mid Cap	0.091	−5.7%
North American Opportunity	0.063	2.5%
American Growth	0.075	−5.8%
Growth Equity	0.085	26.4%
International Equity	0.069	−6.7%

2.3 Ratios, Proportions, and Prorating

Mechanics

For the following four questions, reduce the ratios to their smallest terms.

126) **a)** $66 : 12$ **b)** $48 : 112 : 80$ **c)** $24 : 36 : 84 : 108$

127) **a)** $7.2 : 6$ **b)** $0.03 : 0.035 : 0.02$ **c)** $0.27 : 0.18 : 0.51 : 0.15$

128) **a)** $5\frac{1}{4} : 6$ **b)** $\frac{3}{2} : \frac{9}{5} : \frac{21}{10}$ **c)** $2\frac{1}{8} : \frac{9}{8} : 3\frac{3}{8} : 1$

129) **a)** $\frac{8}{3} : \frac{5}{9}$ **b)** $\frac{5}{6} : \frac{1}{3} : 2$ **c)** $\frac{1}{7} : \frac{1}{3} : \frac{1}{4} : \frac{1}{6}$

130) Reduce the following ratios to the smallest term of one. Round to two decimals as needed.

   a) $48 : 53$ **b)** $5\frac{7}{8} : 2\frac{3}{4} : 3\frac{1}{12}$ **c)** $\frac{2}{3} : 11\frac{1}{11} : 5\frac{5}{9} : 2.08$

131) Solve the following proportions for the unknown variable. Round to two decimals as needed.

   a) $7 : 12 = 109 : y$

   b) $3.23 : 4.07 : 2.12 = x : 55.9625 : 29.15$

   c) $q : \frac{5}{3} = 13.75 : 25$

132) Solve the following proportions for all of the unknown variables.

   a) $12.15 : 38.30 : r = x : 59{,}184.53 : 26{,}998.93$

   b) $16 : 5 : 9 : 30 = g : h : i : 397{,}767$

For the following two questions, prorate the total as indicated by the ratio.

133) $Ratio = 3 : 2, Total = \$11{,}368.25$

134) $Ratio = 7 : 5 : 3, Total = \$46{,}923.90$

Applications

135) EB Games sells gaming consoles. Last month, it sold $22,500 worth of Nintendo Wii consoles, $31,500 worth of Microsoft Xbox consoles, and $18,000 worth of Sony PlayStation consoles. Express the ratio of Wii to Xbox to PlayStation sales in its lowest terms.

136) The manufacturing cost of a deluxe candle is made up of $2.40 in paraffin, $1.20 in dye, $1.60 in overhead, and $4.40 in direct labour. Express the ratio between these costs respectively in their lowest terms.

137) If one Canadian dollar buys $1.0385 of US dollars, how much US money can you spend so that you do not exceed the maximum duty-free amount of $800 Canadian on your next US vacation?

138) For every $100 of retail spending, the average Canadian spends approximately $19.12 at motor vehicle dealerships and $18.35 at grocery stores. If the average Canadian spends $49,766 per year on retail spending, how much more does a Canadian spend annually at motor vehicle dealerships than at grocery stores?

139) Marina has a three-sevenths interest in a partnership. If she sells one-quarter of her interest for $8,250, what is the implied value of the partnership?

140) You are making a punch for an upcoming party. The recipe calls for 2½ parts ginger ale to ⅘ parts grenadine to 1⅜ parts vodka. If your punch bowl can hold 8.5 litres, how much (in litres) of each ingredient is needed? Round the answers to two decimals as needed.

Challenge, Critical Thinking, and Other Applications

141) A manufacturing facility requires one member of the board of directors for every 10 executives. There are five managers for every executive and eight workers for every manager. The average salary paid to directors is $105,000 along with $80,000 to executives, $55,000 to managers, and $30,000 to workers. If the facility has 1,383 employees, what is the total labour cost?

142) It is common for strip mall owners to allocate to their tenants general overhead costs such as snow clearing, security, and parking lot maintenance in one of two ways. The first method involves allocating these costs on the basis of the square metres each tenant operates. The second method involves allocating these costs on the basis of the number of annual transactions made by each tenant. At Charleswood Square, there are three tenants. The 7-Eleven takes up 150 square metres, Tim Hortons takes up 235 square metres, and Quiznos takes up 110 square metres. Last year, 7-Eleven had 350,400 transactions, Tim Hortons had 657,000 transactions, and Quiznos had 197,100 transactions. Total overhead costs for Charleswood Square last year were $25,980. Determine which method of allocation each tenant would prefer and how much of a savings in percentage its preferred method would represent relative to the other allocation method.

143) The average annual Canadian cellphone plan covers 780 minutes of voice time, 600 text messages, and 8 video messages with a total annual cost of $500.63. However, some Canadians are high-usage consumers and average 439 minutes per month.

    a) Assuming the same proportion of usage and that each component shares the cost equally on a per-unit basis, how many text messages and video messages do these

high-usage Canadian consumers average per year? How much should they pay annually for their cellphone usage?

b) A similar typical cellphone plan in the Netherlands has a total annual cost of $131.44. How much would a high-usage plan cost annually in the Netherlands?

144) Procter & Gamble (P&G) is analyzing its Canadian sales by region: 23% of sales were on the West coast; 18% were in the Prairies, 45% were in central Canada, and 14% were in Atlantic Canada. Express and reduce the ratio of P&G's respective sales to the smallest term of one. Round to two decimals.

145) A company has 14 managers, 63 supervisors, and 281 workers. After a record profit year, the company wants to distribute an end-of-year bonus to all employees. Each worker is to get one share, each supervisor gets twice as many shares as each worker, and each manager gets twice as many shares as each supervisor. If the bonus to be distributed totals $1,275,000, how much does each manager, supervisor, and worker receive?

Chapter 3 Human Resources and Economic Applications

3.1 Gross Earnings

Mechanics

146) Laars earns an annual salary of $60,000. Determine his gross earnings per pay period under each of the following payment frequencies:
a) Monthly **b)** Semi-monthly **c)** Biweekly **d)** Weekly

147) A worker earning $13.66 per hour works 47 hours in the first week and 42 hours in the second week. What are his total biweekly earnings if his regular workweek is 40 hours and all overtime is paid at 1.5 times his regular hourly rate?

148) Marley is an independent sales agent. He receives a straight commission of 15% on all sales from his suppliers. If Marley averages semi-monthly sales of $16,000, what are his total annual gross earnings?

149) Sheila is a life insurance agent. Her company pays her based on the annual premiums of the customers that purchase life insurance policies. In the last month, Sheila's new customers purchased policies worth $35,550 annually. If she receives 10% commission on the first $10,000 of premiums and 20% on the rest, what are her total gross earnings for the month?

150) Tuan is a telemarketer who earns $9.00 per hour plus 3.25% on any sales above $1,000 in any given week. If Tuan works 35 regular hours and sells $5,715, what are his gross earnings for the week?

151) Adolfo packs fruit in cans on a production line. He is paid a minimum wage of $9.10 per hour and earns $0.09 for every can packed. If Adolfo manages to average 160 cans per hour, what are his total gross earnings daily for an eight-hour shift?

Applications

152) Charles earns an annual salary of $72,100 paid biweekly based on a regular workweek of 36.25 hours. His company generously pays all overtime at twice his regular wage. If Charles worked 85.5 hours over the course of two weeks, what are his gross earnings for that period?

153) In order to motivate a manufacturer's agent to increase his sales, a manufacturer offers monthly commissions of 1.2% on the first $125,000, 1.6% on the next $150,000, 2.25% on the next $125,000, and 3.75% on anything above. If the agent managed to sell $732,000 in a single month, what commission is he owed?

154) Humphrey and Charlotte are both sales representatives for a pharmaceutical company. In a single month, Humphrey received $5,545 in total gross earnings while Charlotte received $6,388 in total gross earnings. In sales dollars, how much more did Charlotte sell if they both received 5% straight commission on their sales?

155) Mayabel is a cherry picker working in the Okanagan Valley. She can pick 17 kg of cherries every hour. The cherries are placed in pails that can hold 13.6 kg of cherries. If she works 40 hours in a single week, what are her total gross earnings if her piecework rate is $17.00 per pail?

156) Miranda is considering three relatively equal job offers and wants to pick the one with the highest gross earnings. The first job is offering a base salary of $1,200 semi-monthly plus 2% commission on monthly sales. The second job offer consists of a 9.75% straight commission. Her final job offer consists of monthly salary of $1,620 plus 2.25% commission on her first $10,000 in monthly sales and 6% on any monthly sales above that amount. From industry publications, she knows that a typical worker can sell $35,000 per month. Which job offer should she choose and, on the annual basis, how much better is it than the other job offers?

157) A Canadian travel agent is paid a flat rate of $37.50 for every vacation booked through a certain airline. If the vacation is in North America, the agent also receives a commission of 2.45%. If the vacation is international, the commission is 4.68%. What are the total monthly gross earnings for the agent if she booked 29 North American vacations worth $53,125 and 17 international vacations worth $61,460?

158) Vladimir's employer has just been purchased by another organization. In the past, he has earned $17.90 per hour and had a normal workweek of 37.5 hours. However, his new company only pays its employees a salary semi-monthly. How much does Vladimir need to earn each paycheque to be in the same financial position?

Challenge, Critical Thinking, and Other Applications

159) An employee on salary just received his biweekly paycheque in the amount of $1,832.05, which included pay for five hours of overtime at time-and-a-half. If a normal workweek is 40 hours, what is the employee's annual salary?

160) A graduated commission scale pays 1.5% on the first $50,000, 2.5% on the next $75,000, and 3.5% on anything above. What level of sales would it take for an employee to receive total gross earnings of $4,130?

161) A sales organization pays a base commission on the first $75,000 in sales, base + 2% on the next $75,000 in sales, and base + 4% on anything above. What is the base commission if an employee received total gross earnings of $7,500 on $200,000 in sales?

162) A typical sales agent for a company has annual sales of $4,560,000, equally spread throughout the year, and receives a straight commission of 2%. As the new human resource specialist, you have been assigned the task to improve employee morale by developing different pay options of equivalent value to offer to the employees. Your first option is to pay them a base salary of $2,000 per month plus commission. Your second option is to pay a base commission monthly on their first $200,000 in sales, and

a base + 2.01% on anything over $200,000 per month. In order to equate all the plans, determine the required commission rates, rounded to two decimals in percent format, in both options.

163) Shaquille earns an annual salary of $28,840.50 paid biweekly. His normal workweek is 36.25 hours and overtime is paid at twice the regular rate. In addition, he is paid a commission of 3% of sales on the first $25,000 and 4% on sales above that amount. What are his total gross earnings during a pay period if he worked 86 hours and had sales of $51,750?

164) Janagan works for Games Galore and gets paid $46,000 in annual base salary, paid out monthly based on the regular workweek of 38 hours per week. He gets paid time-and-a-half when he works overtime. He receives a 2% commission on sales over $2500 during each pay period. How many hours of overtime did he work during the pay period in which he made $2850 in sales and earned $4014.93 in total compensation?

165) Janagan works for Games Galore and gets paid monthly based on the regular workweek of 35 hours per week. He gets paid time-and-a-half when he works overtime. His base annual salary is $40,000, and he receives a 1% commission on sales over $1500 during each pay period. How much did he make in sales during the pay period in which he worked 6 hours of overtime and earned $3541.94 in total compensation?

3.2 Personal Income Tax

Use the provincial and federal tax tables for 2020 in this textbook section when you determine income taxes in the following exercises.

Mechanics

For the following three questions, determine only the federal income taxes on the given gross incomes.

166) $22,375

167) $158,914

168) $102,100

For the following three questions, determine only the provincial income taxes on the given gross incomes.

169) $61,000 in Newfoundland and Labrador

170) $83,345 in Saskatchewan

171) $78,775 in British Columbia

For the following two questions, determine the total federal and provincial/territorial taxes on the given gross incomes.

172) $48,910 in the Northwest Territories

173) $65,525 in Prince Edward Island

Applications

174) Nadia lives in Manitoba and has three part-time jobs. Her gross annual income from each job was $5,300, $21,450, and $25,390. How much federal and provincial income tax does Nadia owe?

175) If Delaney's gross income increases from $71,000 to $79,000, by what dollar amount will her after-tax pay change once federal and provincial income taxes are deducted in the province of Quebec?

176) Helen just moved from Saskatchewan to her new job in Alberta at the same rate of pay of $51,225. By what dollar amount will her provincial income taxes change?

177) Jane has received two job offers that she thinks are relatively equal. The only difference lies in the salary. The first job offer is for a position in Nova Scotia earning gross income of $63,375. The second job offer is for a position in British Columbia earning gross income of $61,990. If Jane will select the job that has the highest after-tax income, which one should she choose? How much better is this option in dollars?

178) Suppose an employee earns $111,300 in gross income. Calculate the federal and provincial income taxes that need to be deducted if he lives in either Saskatchewan, Ontario, or the Northwest Territories. In which province or territory will he earn the most income after income tax deductions? The least? What is the dollar amount difference among the three alternatives?

179) After both federal and provincial income taxes are deducted, who would earn more money, an individual earning $85,000 in New Brunswick or an individual earning $79,000 in Nunavut?

Challenge, Critical Thinking, & Other Applications

180) Rawatha is the human resources manager for her firm located in Yukon. The salespeople for her organization are paid an annual base salary of $30,000 plus 8% straight commission on their annual sales. If the average salesperson sells $210,000 annually, what is the total annual federal and provincial income tax owing on the salesperson's gross income?

181) Esmerelda works on the production line and is paid a piecework rate of $2.25 per unit produced. She is capable of producing an average of five units per hour, and works eight hours every weekday, five days per week. Assuming a 52-week year, what is the annual federal income tax and provincial income tax owing if she lives in Prince Edward Island?

182) Mary Jane is paid biweekly at a rate of $34.68 per hour and works 36.25 hours every week. Assuming a 52-week year, what are the amounts of the federal income tax deduction and provincial income tax deduction that must be removed from each pay cheque if Mary Jane lives in Manitoba? (Hint: income tax would be deducted equally across all pay cheques for the year.)

183) Felix was transferred from his head office in Ontario to Alberta. In accepting the transfer, his employer agreed to increase his salary of $88,000 by $7,000. What is the percent change in Felix's after-tax income?

184) If a wage earner paid $2,277.98 in total annual Ontario provincial income taxes, determine his annual gross income.

185) Perform a comparison across all territories and provinces of the income earned after both federal and provincial income taxes are deducted for a Canadian who earns $56,738 in gross income. Which province or territory has the highest income after deductions? Which has the lowest? What is the percent difference between the two?

3.3 Indexes

Mechanics

For the following four questions, solve for the unknown variables (identified with a question mark) based on the information provided. Round index numbers to one decimal and base values to the nearest integer.

	Chosen Quantity ($)	Base Quantity ($)	Base Value	Index Number
186)	$559.99	$539.99	100	?
187)	$315,000.00	?	1,000	12,415.9
188)	$114.30	$112.50	?	101.6
189)	?	$248.75	1,000	1,548.9

For following five questions, solve for the unknown variables (identified with a question mark) based on the information provided.

	Consumer Price Index (CPI)	Purchasing Power of $1 (PPD)	
190)	107.9	?	
191)	?	$0.80	
	Nominal Income ($)	**Consumer Price Index (CPI)**	**Real Income (RI)**
192)	$45,000.00	120.0	?
193)	?	105.0	$29,523.81
194)	$86,000.00	?	$80,298.71

Applications

195) In Regina, a 4 L bag of 2% milk has an average price of $3.71. Prices in Toronto and Montreal are $4.55 and $5.40, respectively. Calculate an index of these prices using Regina as the base.

196) If Sabrina is currently earning $53,000 and the CPI changes from 105.9 to 107.6, how much money does she need to earn next year just to keep up with inflation?

197) If the CPI rises from 103.4 to 108.8, how much money at the start is required to have the same purchasing power as $100 at the end?

198) George currently earns $28,000 per year. As per his union salary grid, next year he will earn $32,500 per year. If the CPI increases from 104.0 to 106.1 over the same time frame, how much of his raise is an actual raise if accounting for inflation?

199) Last year the purchasing power of a dollar was 84.3%. This year the purchasing power of a dollar is 81.4%. What is the percent change in the CPI between the two years?

Challenge, Critical Thinking, and Other Applications

200) An investor had $200,000 invested in an S&P/TSX Composite Index portfolio in 2009. By 2011, the index rose to 14,136.50. If the index in 2009 was 8,694.90, how much money would the investor have in her portfolio in 2011, if she was able to match the market performance?

201) Over the years, Hannah's income has changed from $36,000 to $40,000 to $45,000. If the CPI changed from 102.9 to 105.1 to 108.6 over the same time period, determine Hannah's percent change in real income from year to year.

202) The CPI rose from 102.6 in 2003 to 116.5 in 2010. The S&P/TSX Composite Index rose from 6,569.49 to 11,294.42 over the same time period. If an investor had $125,000 invested in 2003, how much of the growth (in dollars) for the portfolio over the seven years represents actual growth if inflation is considered?

203) An enterprise has 136 employees earning an average income of $42,250 per year. If the CPI rises from 103.7 to 107.2, by what amount do total wages rise if all employee wages are adjusted to match the CPI?

204) Using January 2003 as your base year with a base value of 100, compute a series of indexes for the S&P/TSX Composite Index from select years in 2003 to 2011. Interpret your results.

Year	2003	2005	2007	2009	2011
January index	6,569.49	9,204.05	13,034.12	8,694.90	13,551.91

205) Below is select information on the CPI from 2004 to 2010:

Year	2004	2006	2008	2010
July CPI	105.0	109.6	115.8	116.8

a) Determine the purchasing power of a dollar for each year. Interpret your results.

b) If you were earning $34,500 in 2002, calculate the nominal income required each year to keep up with the changes in the CPI.

Chapter 4 Marketing Applications

4.1 Discounts

Round all money to two decimals and percentages to four decimals in each of the following questions.

Mechanics

For the following four questions, solve for the unknown variables (identified with a ?) based on the information provided. "N/A" indicates that the particular variable is not applicable in the question.

	List Price or MRSP	First Discount	Second Discount	Third Discount	Net Price	Equivalent Single Discount Rate	Total Discount Amount
206)	$980.00	42%	N/A	N/A	?	N/A	?
207)	?	25%	N/A	N/A	$600.00	N/A	?
208)	$1,975.00	25%	15%	10%	?	?	?
209)	?	18%	4%	7%	$366.05	?	?

Applications

210) A wholesaler of stereos normally qualifies for a 35% trade discount on all electronic products purchased from its manufacturer. If the MSRP of a stereo is $399.95, what net price will the wholesaler pay?

211) Mary is shopping at the mall where she sees a sign that reads, "Everything in the store is 30% off, including sale items!" She wanders in and finds a blouse on the clearance rack. A sign on the clearance rack states, "All clearance items are 50% off." If the blouse is normally priced at $69.49, what price should Mary pay for it?

212) A distributor sells some shoes directly to a retailer. The retailer pays $16.31 for a pair of shoes that has a list price of $23.98. What trade discount percent is the distributor offering to its retailers?

213) A retailer purchases supplies for its head office. If the retailer pays $16.99 for a box of paper and was eligible for a 15% volume discount, what was the original MSRP for the box of paper?

214) Mountain Equipment Co-op has purchased a college backpack for $29 after discounts of 30%, 8%, and 13%. What is the MSRP for the backpack? What single discount is equivalent to the three discounts?

215) Walmart purchased the latest CD recorded by Selena Gomez. It received a total discount of $10.08 off the MSRP for the CD, which represents a discount percent of 42%.

   a) What was the MSRP?

   b) What was the net price paid for the CD?

216) Best Buy just acquired an HP Pavilion computer for its electronics department. The net price on the computer is $260.40 and Best Buy receives discounts of 40% and 38%.

    a) What single discount is equivalent to the two discounts?

    b) What is the list price?

    c) What is the total discount amount?

217) TELUS retails a Samsung cellphone at the MSRP of $399.99. TELUS can purchase the phone from its supplier and receive a 20% trade discount along with a 5% volume discount.

    a) What is the single equivalent percent discount?

    b) What net price does TELUS pay for the phone?

    c) How much of a discount in dollars does this represent?

218) A wholesaler offers the following discounts: 10% seasonal discount for all purchases made between March 1 and May 1, 15% cumulative quantity discount whenever more than 5,000 units are purchased in any month, 5% loyalty discount for customers who have made regular purchases every month for at least one year, and a 33% trade discount to any retailer. Ed's Retail Superstore makes a purchase of 200 watches, MSRP $10, from the wholesaler on April 29. This month alone, Ed's has ordered more than 5,000 watches. However, Ed's has purchased from the wholesaler for only the past six months. Determine the total price that Ed's should pay for the watches.

219) If a distributor is eligible for a 60% trade discount, 5% volume discount, and 3% seasonal discount, what single equivalent discount rate would it be eligible to receive? If the trade discount is applied first and equals a trade discount of $48, calculate the net price for the item.

Challenge, Critical Thinking, and Other Applications

220) A human resource manager needs to trim labour costs in the following year by 3%. If current year labour costs are $1,231,498, what are the labour costs next year?

221) At an accounting firm, the number of accountants employed is based on the ratio of 1:400 daily manual journal entries. Because of ongoing increases in automation, the number of manual journal entries declines at a constant rate of 4% per year. If current entries are 4,000 per year, how many years and days will it take until the firm needs to lay off one accountant? (Hint: An accountant is laid off when the number of journal entries drops below 3,600.)

222) An economist is attempting to understand how Canada reduced its national debt from 1999 to 2008. In 1999, Canada's national debt was $554.143 billion. In 2008, the national debt stood at $457.637 billion. What percentage had the national debt been reduced by during this time period?

223) Sk8 is examining an invoice. The list price of a skateboard is $109.00, and the invoice states it received a trade discount of 15% and quantity discount of 10% as well as a loyalty discount. However, the amount of the loyalty discount is unspecified.

    a) If Sk8 paid $80.88 for the skateboard, what is the loyalty discount percent?

    b) If the loyalty discount is applied after all other discounts, what amount of loyalty dollars does Sk8 save per skateboard?

224) Currently, a student can qualify for up to six different tuition discounts at a local college based on such factors as financial need or corporate sponsorships. Mary Watson just applied to the college and qualifies for all six discounts: 20%, 15%, 23%, 5%, 3%, and 1%.

    a) She is confused and wants the college to tell her what single discount percent she is receiving. What should the college tell her?

    b) If her total list tuition comes to $6,435.00, how much should she pay?

225) Sumandeep is very loyal to her local hairstylist. Because she is loyal, her hairstylist gives her three different discounts: 10%, 5%, and 5%. These discounts amount to $14.08 in savings.

    a) What was the list price her hairstylist charged her?

    b) What amount did she pay her hairstylist?

    c) If her hairstylist increases prices by 5%, what are the list price, net price, and total discount amount?

4.2 Markup

Round all money to two decimals and percentages to four decimals for each of the following exercises.

Mechanics

For the following eight questions, solve for the unknown variables (identified with a ?) based on the information provided.

	Regular Unit Selling Price	Cost	Expenses	Profit	Markup Amount	Break-Even Price	Markup on Cost	Markup on Selling Price
226)	?	$188.42	$48.53	$85.00	?	?	?	?
227)	$999.99	?	30% of C	23% of C	?	?	?	?
228)	?	?	?	10% of S	$183.28	?	155%	?
229)	$274.99	?	20% of S	?	?	?	?	35%
230)	?	?	45% of C	?	$540.00	$1,080.00	?	?
231)	?	$200 less 40%	?	15% of S	?	?	68%	?
232)	?	?	$100.00	?	$275.00	?	?	19%
233)	?	?	15% of C	12% of S	?	$253.00	?	?

Applications

234) If a pair of sunglasses sells at a regular unit selling price of $249.99 and the markup is always 55% of the regular unit selling price, what is the cost of the sunglasses?

235) A transit company wants to establish an easy way to calculate its transit fares. It has determined that the cost of a transit ride is $1.00, with expenses of 50% of cost. It requires $0.75 profit per ride. What is its markup on cost percentage?

236) Daisy is trying to figure out how much negotiating room she has in purchasing a new car. The car has an MSRP of $34,995.99. She has learned from an industry insider that most car dealerships have a 20% markup on selling price. What does she estimate the dealership paid for the car?

237) The markup amount on an eMachines desktop computer is $131.64. If the machine regularly retails for $497.25 and expenses average 15% of the selling price, what profit will be earned?

238) Manitoba Telecom Services (MTS) purchases an iPhone for $749.99 less discounts of 25% and 15%. MTS's expenses are known to average 30% of the regular unit selling price.

    a) What is the regular unit selling price if a profit of $35 per iPhone is required?

    b) What are the expenses?

    c) What is the markup on cost percentage?

    d) What is the break-even selling price?

239) A snowboard has a cost of $79.10, expenses of $22.85, and profit of $18.00.

    a) What is the regular unit selling price?

    b) What is the markup amount?

    c) What is the markup on cost percentage?

    d) What is the markup on selling price percentage?

    e) What is the break-even selling price? What is the markup on cost percentage at this break-even price?

Challenge, Critical Thinking, & Other Applications

240) A waterpark wants to understand its pricing better. If the regular price of admission is $49.95, expenses are 20% of cost, and the profit is 30% of the regular unit selling price, what is the markup amount?

241) Sally works for a skateboard shop. The company just purchased a skateboard for $89.00 less discounts of 22%, 15%, and 5%. The company has standard expenses of 37% of cost and desires a profit of 25% of the regular unit selling price. What regular unit selling price should Sally set for the skateboard?

242) If an item has a 75% markup on cost, what is its markup on selling price percentage?

243) A product received discounts of 33%, 25%, and 5%. A markup on cost of 50% was then applied to arrive at the regular unit selling price of $349.50. What was the original list price for the product?

244) Mountain Equipment Co-op (MEC) wants to price a new backpack. The backpack can be purchased for a list price of $59.95 less a trade discount of 25% and a quantity discount of 10%. MEC estimates expenses to be 18% of cost and it must maintain a markup on selling price of 35%.

  a) What is the cost of backpack?

  b) What is the markup amount?

  c) What is the regular unit selling price for the backpack?

  d) What profit will Mountain Equipment Co-op realize?

  e) What happens to the profits if it sells the backpack at the MSRP instead?

245) Costco can purchase a bag of Starbucks coffee for $20.00 less discounts of 20%, 15%, and 7%. It then adds a 40% markup on cost. Expenses are known to be 25% of the regular unit selling price.

  a) What is the cost of the coffee?

  b) What is the regular unit selling price?

  c) How much profit will Costco make on a bag of Starbucks coffee?

  d) What markup on selling price percentage does this represent?

  e) Repeat questions (a) through (d) if the list price changes to $24.00.

4.3 Markdown

Round all money to two decimals and percentages to four decimals for each of the following exercises.

Mechanics

For the following six questions, solve for the unknown variables (identified with a ?) based on the information provided.

	Regular Selling Price	Markdown Amount	Markdown Rate	Sale Price
246)	$439.85	?	35%	?
247)	?	$100.00	?	$199.95
248)	$1,050.00	?	?	$775.00
249)	$28,775.00	$3,250.00	?	?
250)	?	?	33%	$13,199.95
251)	?	$38.33	12%	?

Applications

252) A pair of Nike athletic shoes is listed at a regular selling price of $89.99. If the shoes go on sale for 40% off, what is the sale price?

253) During its special Bay Days, The Bay advertises a Timex watch for $39.99 with a regular price of $84.99. Calculate the markdown rate and markdown amount.

254) For spring break you are thinking about heading to Tulum, Mexico. In planning ahead, you notice that a one-week stay at the Gran Bahia Principe Tulum, regularly priced at $2,349 for air and six nights all inclusive, offers an early-bird booking discount of $350. What markdown rate is being offered for booking early?

255) A Heritage Infusio deep frying pan is advertised at 70% off with a sale price of $39.99. What is the frying pan's regular selling price, and what markdown amount does this represent?

256) A mass merchandiser uses its Lagostina cookware product line as a marketing tool. The cookware is always on sale at an advertised price of 45% off. The cost of the cookware is $199.99, expenses are $75, and the planned profit at the sale price is $110. Calculate the sale price and the advertised price for the cookware.

257) Quicky Mart regularly sells its Red Bull sports drink for $2.99 per can. Quicky Mart noticed that one of its competitors down the street sells Red Bull for $1.89. What markdown rate must Quicky Mart advertise if it wants to match its competitor?

258) A hardware store always advertises a Masterdesigner 75-piece screwdriver set at 80% off for a sale price of $17.99.

   a) If the cost of the set is $10 and expenses are 30% of the sale price, what is the planned profit when the product is on sale?

   b) What profit is earned if the product actually sells at its regular selling price?

259) A campus food outlet is advertising a "Buy one, get one 25% off" deal. The 25% off comes off the lower-priced item. If you purchase a chicken dinner for $8.99 and your friend gets the burger combo for $6.99, what is the markdown rate on the total price?

260) Blast'em Stereos purchases a stereo system for $1,900 less two discounts of 40% and 18%. The store uses this product to draw customers to the store and always offers the stereo on sale at 25% off. When the stereo is on sale, it plans on expenses equaling 30% of the cost and a profit of 20% of the sale price.

   a) What is the sale price for the stereo?

   b) How much profit does Blast'em make when the stereo sells at the sale price?

   c) By law, this stereo must sell at the regular selling price for a period of time before going on sale. What is the regular selling price?

   d) What profit does Blast'em earn if a customer purchases the stereo during this initial period?

Challenge, Critical Thinking, & Other Applications

261) Frigid Boards purchases one of its snowboards for $395 less a retail trade discount of 15% and a loyalty discount of 4%. Its markup on selling price percentage on all snowboards is 21%. At the end of the season, any leftover snowboards are marked down by 10%. What is the sale price for the snowboard?

262) An HP LaserJet printer has an MSRP of $399.95. It is subject to trade discounts of 30% and 23%. The LaserJet is a featured item for a computer store and is always on sale. The store plans to sell the LaserJet for a sale price that allows it to cover expenses equaling 15% of cost and realize a profit of $35.00.

a) What is the sale price?

b) If the MSRP is the regular unit price of the printer, what rate of markdown can the computer store advertise?

c) What markup rate on the selling price is realized at the sale price?

263) The Brick advertises that when you purchase a queen-size Tempur-Pedic mattress set for $2,499.97 it will give you a 51″ 3-D plasma television with a 3-D starter kit included. The value of this gift is $1,199.99. What markdown rate does this represent?

264) A Maytag 27 cubic foot refrigerator retails for $2,400.00 at Landover Appliance Centre. The company, which is celebrating its 30th anniversary this coming weekend, features the fridge for 30% off. The markup rate on the selling price of the fridge at the regular unit selling price is 53%.

a) What is the sale price?

b) At the sale price, what is the markup rate on selling price?

c) If the expenses are 15% of the regular selling price, what is the profit when the fridge is on sale?

265) Dreger Jewelers is selling a diamond bracelet. It uses this bracelet in its promotions and almost always has it on sale. The cost of the bracelet is $2,135 less discounts of 20% and 30%. When the bracelet is on sale for 25% off, the expenses are 15% of cost and the profit is 20% of cost.

a) What is the sale price?

b) What is the bracelet's regular selling price?

c) If the bracelet sells at the regular selling price, what are the markup amount and the markup rate on cost?

Chapter 5 Accounting Applications

5.1 Sales Taxes

Mechanics

You are purchasing a new BlackBerry at the MSRP of $649.99. Calculate the price including taxes in the following provinces or territories:

266) Northwest Territories

267) New Brunswick

268) Nova Scotia

269) British Columbia

The Brick is advertising a new Serta mattress nationally for a price of $899.99 including taxes. What is the price before taxes and the sales tax amounts in each of the following provinces?

270) Ontario

271) Saskatchewan

272) Audiophonic Electronics is calculating its HST remittance in Prince Edward Island. For each of the following months, calculate the HST remittance or refund on these HST-eligible amounts.

Month	Purchases	Sales
January	$48,693	$94,288
February	$71,997	$53,639

273) Airwaves Mobility is calculating its GST remittance in Alberta. For each of the following quarters, calculate the GST remittance or refund on these GST-eligible amounts.

Quarter	Purchases	Sales
Winter	$123,698	$267,122
Spring	$179,410	$158,905
Summer	$216,045	$412,111
Fall	$198,836	$175,003

Applications

274) Elena lives in Nova Scotia and has relatives in Alberta, Saskatchewan, and Quebec. She gets together with them often. She wants to purchase a new aerobic trainer and would like to pay the lowest price. If a family member buys the item, Elena can pick it up at one of their regular family gatherings. The price of the trainer for each province is listed below:

Province	Regular Selling Price before Taxes
Nova Scotia	$1,229.50
Alberta	$1,329.95
Saskatchewan	$1,274.25
Quebec	$1,219.75

a) Where should Elena have the aerobic trainer purchased and how much would she pay?

b) How much money would she save from her most expensive option?

275) Mary Lou just purchased a new digital camera in Nunavut for $556.49 including taxes. What was the price of the camera before taxes? What amount of sales tax is paid?

276) Marley is at Peoples Jewellers in New Brunswick wanting to purchase an engagement ring for his girlfriend. The price of the ring is $2,699.95. If the credit limit on his credit card is $3,000, will he be able to purchase the ring on his credit card? If not, what is the minimum amount of cash that he must put down to use his credit card?

277) In the IKEA store in Vancouver, British Columbia, you are considering the purchase of a set of kitchen cabinets priced at $3,997.59. Calculate the amount of GST and PST you must pay for the cabinets, along with the total price including taxes.

278) A company in Saskatchewan recorded the following GST-eligible purchases and sales throughout the year. Determine the GST remittance or refund per quarter.

Quarter	Purchases	Sales
1st	$2,164,700	$2,522,000
2nd	$1,571,300	$2,278,700
3rd	$1,816,100	$1,654,000
4th	$2,395,900	$1,911,700

279) A manufacturer in Nova Scotia recorded the following HST-eligible purchases and sales in its first three months of its fiscal year. Determine the HST remittance or refund per month.

Month	Purchases	Sales
March	$20,209	$26,550
April	$28,861	$20,480
May	$22,649	$42,340

Challenge, Critical Thinking, and Other Applications

280) If the selling price of an item is 6% higher in Yukon than in Ontario, will the price including taxes be higher in Yukon or Ontario? What percentage more?

281) Colin just travelled across the country on a road trip. He bought some skis in Alberta for $879.95 plus tax, a boombox in British Columbia for $145.58 including taxes, a Niagara Falls souvenir in Ontario for $99.97 plus tax, and some maple syrup in Quebec for $45.14 including tax. Overall, how much GST, PST, and HST did Colin pay on his trip?

282) Cisco Enterprises in Ontario purchased the following in a single month:

16,000 units of network routers at $79.25 each, priced at $97.97 each

12,000 units of wireless LAN adapters at $129.95 each, priced at $189.55 each

13,500 units of computer boards at $229.15 each, priced at $369.50 each.

Assuming that all units purchased are sold during the same month and that all purchases and sales are taxable, calculate the tax remittance or refund for the month.

283) In Quebec, the PST used to be calculated on the price including GST. When the PST was calculated in this manner, what PST rate did Quebec set to arrive at the same price including taxes?

284) For each of the following situations, compute the selling price of the product before taxes in the other province/territory that would result in the same selling price including taxes as the item listed.

	Price before tax	Sold in	Find equivalent price before tax in
a)	$363.75	British Columbia	Prince Edward Island
b)	$1,795.00	Alberta	Manitoba
c)	$19,995.95	Saskatchewan	Ontario
d)	$4,819.35	New Brunswick	Quebec

285) A company made the following taxable transactions in a single month. Compute the GST remittance on its operations assuming all sales and purchases are eligible for GST.

Transaction Type	Unit Price	Quantity Involved
Purchase	$168.70	5,430
Sale	$130.00	4,000
Sale	$148.39	3,600
Purchase	$93.47	2,950
Purchase	$24.23	3,325
Purchase	$121.20	2,770
Sale	$188.88	6,250

5.2 Property Taxes

Mechanics

For the following four questions, solve for the unknown variables (identified with a question mark) based on the information provided.

	Market Value	Tax Policy	Assessed Value	Rate	Type of Rate	Property Tax
286)	$320,000	55%	?	26.8145	Mill	?
287)	?	85%	$136,000	1.984561	Percent	?
288)	$500,000	?	?	9.1652	Mill	$3,666.08
289)	?	50%	$650,000	?	Coefficient	$4,392.91

Applications

290) A house with an assessed value of $375,000 is subject to a tax rate of 1.397645%. What is the property tax?

291) If a commercial railway property has a property tax bill of $166,950 and the mill rate is 18.5500, what is the assessed value of the property?

292) A house in Calgary has a market value of $450,000. The tax policy is 100%. The property is subject to a 2.6402 mill rate from the City of Calgary and a 2.3599 mill rate from the province of Alberta. What are the total property taxes?

293) A residential property in Regina has a market value of $210,000. The Saskatchewan tax policy is 70%. The property is subject to three mill rates: 13.4420 in municipal taxes, 1.4967 in library taxes, and 10.0800 in school taxes. What amount of tax is collected for each, and what are the total property taxes?

294) A municipality needs to increase its operating budget. Currently, the assessed value of all properties in its municipality total $1.3555 billion and the tax rate is set at 0.976513%. If the municipality needs an additional $1.8 million next year, what tax rate should it set assuming the assessed values remain constant?

295) A municipality set its new mill rate to 10.2967, which increased its total operating budget by $10 million on a constant assessed value of $7.67 billion. What was last year's mill rate?

Challenge, Critical Thinking, and Other Applications

296) A school board is determining the mill rate to set for next year. The assessed property values for next year total $5.782035 billion, representing an increase of 5% over the current year. If the school board needs an additional $5.4 million in funding next year, by what amount should it change its current year mill rate of 11.9985?

297) In the current year, the market value of properties totals $6.896 billion. The current tax policy is 85% and the current mill rate is 15.6712. If the municipality requires an additional $2 million in its operating budget next year, market values increase by 4%, and the tax policy changes to 90%, what mill rate should it set for next year?

298) A $600,000 market value property is assessed with a tax policy of 75% and subject to two mill rates. If the total property taxes are $6,766.67 and the second mill rate is half of the first tax rate, calculate each mill rate.

299) Two properties in different provinces pay the same property taxes of $2,840. One province uses a mill rate of 24.6119 with a 60% tax policy, while the other province uses a tax rate of 1.977442% with an 80% tax policy. Compute the market values for each of these properties.

300) A water utility funded through property taxes requires $900 million annually to operate. It has forecasted increases in its operating costs of 7% and 3.5% over the next two years. Currently, properties in its area have a market value of $234.85 billion, with projected annual increases of 3% and 5% over the next two years. The provincial government has tabled a bill that might change the tax policy from 70% to 75% effective next year, but it is unclear if the bill will pass at this point. For planning purposes, the utility wants to forecast its new mill rates for the next two years under either tax policy. Perform the necessary calculations for the utility.

5.3 Exchange Rates and Currency Exchange

For the following six questions, use the mid-rates in the cross-rate table below to convert the current currency to the desired currency.

	Per C$	Per US$	Per €	Per ¥	Per MXN$	Per AU$
Canadian Dollar (C$)	——	0.9787	1.4012	0.0122	0.0823	1.0360
US Dollar (US$)	1.0218	——	1.4317	0.0125	0.0841	1.0585
Euro (€)	0.7137	0.6985	——	0.0087	0.0588	0.7394
Japanese Yen (¥)	82.0233	80.2765	114.9287	——	6.7540	84.9747
Mexican Peso (MXN$)	12.1445	11.8859	17.0165	0.1481	——	12.5814
Australian Dollar (AU$)	0.9652	0.9447	1.3525	0.0118	0.0795	——

Mechanics

	Current Currency	Desired Currency
301)	C$68,000	US$
302)	¥15,000,000	€
303)	AU$3,000	MXN$
304)	US$180,000	AU$
305)	€230,500	C$
306)	MXN$1,300,000	¥

Applications

307) If the exchange rate is ¥95.3422 per C$, what is the exchange rate for C$ per ¥?

308) Procter & Gamble just received payment for a large export of Tide in the amount of 275,000 Denmark kroner (DKK). If the exchange mid-rate is C$0.1992 per DKK, and the bank charges 3% on its buy rates, how many Canadian dollars will Procter & Gamble receive?

309) The exchange rate per US$ is C$0.9863. The exchange rate per US$ is C$0.9863. If the Canadian dollar depreciates by $0.0421 per US$, how many more or less US$ is C$12,500 able to purchase?

310) Jack is heading home to visit his family in Great Britain and decided to stop at the airport kiosk to convert his money. He needs to convert C$5,000 to British pounds (£). The exchange mid-rate per C$ is £0.5841. The kiosk charges a commission of 4.5% on the conversion, plus a flat fee of £5.00.

    a) How many pounds will Jack receive?

b) What is the percentage cost of his transaction?

311) Yarianni is heading on a vacation. She converts her C$4,000 into Chinese yuan renminbi (CNY) at a sell rate of CNY6.3802 per C$. While in China, she spends CNY14,000 of her money. At the airport, she converts her remaining money into Indian rupees (INR) at a sell rate of INR6.7803 per CNY. In India, she spends INR50,000. When she returns home, she converts her INR back to C$ at a buy rate of C$0.0231 per INR. How many Canadian dollars did she return with? Note that all currencies involved have two decimals.

312) Elena is an international investor. Four years ago she purchased 2,700 shares of a US firm at a price of US$23.11 per share when the exchange rate was US$0.7536 per C$. Today, she sold those shares at a price of US$19.87 per share when the exchange rate was US$1.0063 per C$. In Canadian dollars, determine how much money Elena earned or lost on her investment.

313) International Traders regularly imports products from Hong Kong. If the exchange rate of C$ per Hong Kong dollar (HKD) is 0.1378 and the Canadian dollar appreciates by HKD0.0128, by what amount would the cost of a HKD1,000,000 purchase increase or decrease in Canadian dollars for International Traders?

314) Brian needs to purchase some Brazilian reals (BRL). He takes C$7,500 to the bank and leaves the bank with BRL12,753.20. If the exchange mid-rate per C$ is 1.7621, determine the sell rate commission percentage (rounded to two decimals) charged by the bank.

Challenge, Critical Thinking, and Other Applications

315) Fernando could purchase a 55″ Samsung HDTV in Winnipeg, Manitoba, for $2,999.99 plus taxes. Alternatively, he could head across the border on Black Friday and shop in Grand Forks, North Dakota, where the same product is selling for US$2,499.99 (plus 5% state sales tax and 1.75% local sales tax) at Best Buy. He estimates he would incur $65 in gasoline and vehicle wear and tear, $130 in accommodations, and $25 in food (all money in US$). He would make all purchases on his credit card, which uses the mid-rate plus 2.5%. When returning across the border, he would have to pay in Canadian dollars the 5% GST on the Canadian value of the HDTV not including taxes. Once home, he can then have the North Dakota government refund all taxes paid on the HDTV through their Canadian sales tax rebate program. For all currency exchanges, assume a mid-rate of US$0.9222 per C$. Which alternative is Fernando's better choice and by how much?

316) The current mid-rate is C$1.5832 per €. Scotiabank has a sell rate of C$1.6196 per € while an airport kiosk has a sell rate of C$1.6544 per € plus a service charge of C$4.75. You need to purchase €800.

a) Calculate the fee percentages charged by each financial institution. Round your answers to one decimal.

b) Rounded to two decimals, what percentage more than Scotiabank is the airport kiosk charging on your purchase?

317) Henri and Fran have retired and are considering two options for a two-month vacation in Europe. Their local Lethbridge travel agent is offering them an all-inclusive package deal at C$7,975 per person. Alternatively, they can book their own flights for C$1,425 per person, stay in Britain at a small apartment averaging £65 per night for 30 days, and then in France for €70 average per night for 30 days. Estimated groceries cost a

total of £250 in Britain and €400 in France. They will need to purchase a Eurail pass for €986 each while they are there. The exchange rates are €0.6808 per C$ and £0.5062 per C$. Which alternative is their cheapest option and by how much in Canadian dollars?

318) A Canadian manufacturer imports three parts from different countries. It assembles the three parts into a finished item that is then exported to the United States. Every transaction always involves 25,000 units. Expenses average $6.25 per unit.

Component	Price per unit	Exchange rate last month per C$	Exchange rate this month per C$
Part A	¥1,500	¥107.9420	¥108.9328
Part B	AU$14.38	AU$1.1319	AU$1.0928
Part C	€10.73	€0.6808	€0.6569
Finished product for export	US$59.45	US$1.0128	US$1.0243

Considering currency fluctuations, calculate the change in the profit month-over-month in Canadian dollars for the Canadian manufacturer.

319) In each of the following situations, convert the old amount to the new amount using the information provided.

	Old Amount	Old Exchange Rate per C$	Exchange Rate Change	New Amount
a)	US$625.00	US$0.9255	C$ appreciated by US$0.0213	? US$
b)	€16,232.00	€0.5839	C$ depreciated by €0.0388	? €
c)	¥156,500	¥93.4598	C$ depreciated by ¥6.2582	? ¥
d)	MXN$136,000	MXN$13.5869	C$ appreciated by MXN$0.4444	? MXN$

320) Compare the following four situations and determine which one would result in the largest sum of money expressed in Canadian funds.

	Amount	Exchange Rate per C$
a)	65,204 Algerian dinars (DZD)	DZD65.5321
b)	1,807,852 Colombian pesos (COP)	COP1,781.1354
c)	3,692 Israeli new shekels (ILS)	ILS3.7672
d)	30,497 Thai baht (THB)	THB30.4208

Chapter 6 Finance

6.1 Exponents, Roots and Logarithms

321) Solve the following exponential equations for a given variable. Round your answers, where appropriate, to four decimals.

a) $5 = t^3$

b) $5 = 3^t$

c) $2^x = 0.3$

d) $x^2 = 0.3$

e) $4 = m^{1.04}$

f) $4 = (1.04)^m$

g) $40 = 10 \cdot 6^t$

h) $40 = 10t^6$

i) $3 \cdot n^{15} = 25$

j) $3 \cdot 15^n = 25$

k) $4(-0.02t)^e = 8.2$

l) $0.4 - 5e^{-2t} = 1.5$

m) $4(3x - 5)^3 = 9$

n) $e^{3k} = 5000$

o) $0.4 - 5(-2t)^3 = 1.5$

p) $\sqrt[5]{x^3} = 18$

q) $\frac{7}{10t^6 + 5} = 2$

r) $\sqrt[3]{(2x + 3)^2} = 5$

Solve the following eight equations for the given variable.

322) $m = \frac{4(3x-5)^{3p}}{5t}$; solve for x

323) $m = \frac{4p(3x-5)^{3p}}{5t}$; solve for p

324) $3P - (P + t)^{0.004} = 0$; solve for t

325) $A = P\left(1 + \frac{r}{m}\right)^{mt}$; solve for r

326) $A = P(1 + i)^n$; solve for i

327) $3P = Pe^{0.0004t}$; solve for t

328) $3P = P\left(1 + \frac{r}{m}\right)^{mt}$; solve for r

329) $A = P\left(1 + \frac{r}{m}\right)^n$; solve for m

330) Solve the following equation for each of the variables involved:

$$m = \frac{(a^2 b)^c + r}{f(z - 1)^k}$$

6.2 Simple and Compound Interest

331) If you placed $2,000 into an investment account earning 3% simple interest, how many months does it take for you to have $2,025 in your account?

332) Cuthbert put $15,000 into a nine-month term deposit earning simple interest. After six months he decided to cash the investment in early, taking a penalty of 1% on his interest rate. If he received $393.75 of interest, what was the original interest rate before the penalty on his term deposit?

333) If you want to earn $1,000 of simple interest at a rate of 7% in a span of five months, how much money must you invest?

334) Jessica decided to invest her $11,000 in two back-to-back three-month term deposits. On the first three-month term, she earned $110 of interest. If she placed both the principal and the interest into the second three-month term deposit and earned $145.82 of interest, how much higher or lower was the interest rate on the second term deposit?

335) You are planning a 16-day African safari to Rwanda to catch a rare glimpse of the 700 remaining mountain gorillas in the world. The estimated cost of this once-in-a-lifetime safari is $15,000 including the tour, permits, lodging, and airfare. Your parents have promised you a $10,000 graduation gift. You intend to save this money for five years in a long-term investment earning 8.3% compounded semi-annually. If the cost of the trip will remains the same, will you have enough money five years from now to pay for your trip?

336) Your investment of $9,000 that you started six years ago earned 7.3% compounded quarterly for the first 3¼ years, followed by 8.2% compounded monthly after that. How much interest has your investment earned so far?

337) Nirdosh borrowed $9,300 4¼ years ago at 6.35% compounded semi-annually. The interest rate changed to 6.5% compounded quarterly 1¾ years ago. What amount of money today is required to pay off this loan?

338) Jason invested $10,000 into his RRSP when he turned 20 years old. Maria invested $10,000 into her RRSP when she turned 35 years old. If both earned 9% compounded semi-annually, what percentage more money (rounded to one decimal) will Jason have than Maria when they both turn 65 years old?

339) Dovetail Industries needs to save $1,000,000 for new production machinery that it expects will be needed six years from today. If money can earn 8.35% compounded monthly, how much money should Dovetail invest today?

340) A debt of $37,000 is owed 21 months from today. If prevailing interest rates are 6.55% compounded quarterly, what amount should the creditor be willing to accept today?

341) Rene wants to invest a lump sum of money today to make a $35,000 down payment on a new home in five years. If he can place his money in an investment that will earn 4.53% compounded quarterly in the first two years followed by 4.76% compounded monthly for the remaining years, how much money does he need to invest today?

342) Amadala owes Nik $3,000 and $4,000, due nine months and two years from today, respectively. If she wants to pay off both debts today, what amount should she pay if money can earn 6% quarterly in the first year and 5.75% monthly in the second year?

343) Your company paid an invoice five months late. If the original invoice was for $6,450 and the amount paid was $6,948.48, what monthly compounded interest rate is your supplier charging on late payments?

344) In a civil lawsuit, a plaintiff was awarded damages of $15,000 plus $4,621.61 in interest for a period of 3¼ years. What quarterly compounded rate of interest was used in the settlement?

345) Muriel just received $4,620.01 including $840.01 of interest as payment in full for a sum of money that was loaned 2 years and 11 months ago. What annual rate of interest was charged on the loan?

346) At what monthly compounded interest rate does it take five years for an investment to double?

347) You just took over another financial adviser's account. The client invested $15,500 at 6.92% compounded monthly and now has $24,980.58. How long has this client had the money invested?

348) A debt of $7,500 is owed. Suppose prevailing interest rates are 4.9% compounded quarterly. How far in advance was the debt paid if the creditor accepted a payment of $6,721.58?

349) Wayne was late in making a $3,500 payment to Dora. If Dora accepted a payment of $3,801.75 and charged 5.59% compounded semi-annually, how late was the payment?

**350)** How long will it take $5,750 to become $10,000 at 6.25% compounded weekly?

Chapter 7 Systems of Equations and Linear Programming

7.2 Inequalities in One Variable

351) Solve each inequality and write the solution set using interval form and graph it.

   a) $2x + 1 < 5$

   b) $-2 - 4x \geq 6$

   c) $\frac{1}{2} - x < \frac{x}{5} + \frac{1}{4}$

   d) $\frac{5+x}{3} \geq -1$

   e) $\frac{3-5x}{3} \leq 0$

352) Use graph solution notation to write the following using single interval, if possible.

   a) $(-\infty, 0) \cup (-\infty, 5)$

   b) $(4, 7) \cap (2, \infty)$

   c) $(-3, \infty) \cap (2, \infty)$

   d) $(-\infty, 4) \cup [4, 5]$

   e) $[3, 6] \cap (5, 7]$

   f) $[-5, \infty) \cup (9, \infty)$

353) Solve the following inequalities and describe the solutions using set, interval and graph notation:

   a) $2(t - 5) > 15 - (4 - 2t)$

   b) $\frac{4}{3}(3x - 2) + \frac{3}{5}(4x - 3) \geq \frac{11}{60} + 3x$

354) Solve each compound inequality. Write the solution set using interval notation and graph it.

   a) $5 > 7 - x$ and $2 + 1.5x < 4$

   b) $5 - x < 4$ and $0.2x - 3 < 1$

c) $\frac{2x-5}{-2} < 2$ and $\frac{2x+1}{3} > 0$

d) $1 - x < 7 + x$ or $4x + 3 > x$

e) $5 + x > 3 - x$ or $2x - 3 > x$

f) $\frac{1}{2}(x + 1) > 3$ or $0 < 7 - x$

g) $1 - \frac{3}{2}x < 4$ and $\frac{1}{4}x - 2 \leq -3$

h) $\frac{3}{5}x - 1 > 2$ and $5 - \frac{2}{5}x \geq 3$

i) $1 \leq 3x - 5 < 8$

j) $-2 < 4x + 9 \leq 16$

k) $-2 \leq 3 - 6x < 22$

l) $-13 < 5 - 9x < 41$

355) Write an inequality to describe the following situation: Kate will buy the jacket provided the cost c is at most $117.

356) A company spends $1200 per day on overhead and labor, and each item they produce costs $5 for materials. If they sell the items for $15 each, how many items will they need to sell each day for their profits to be positive?

357) Consider the following rectangle. For what values of x (measured in cm) will the rectangle have the perimeter of at least 242 cm?

$6x + 9$

$3x + 4$
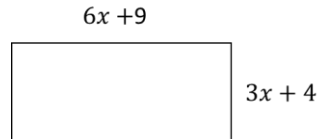
358) For what values of x is each of the following expressions a real number?

a) $\sqrt{6x - 9}$

b) $\frac{5}{x+1}$

c) $\frac{6}{\sqrt{18 - 3x}}$

359) A student scored 67 points on his Psychology 101 midterm. If the average of his midterm and final must be between 79 and 90 inclusive for a B, then for what range of scores on the final exam would a student get a B? Both tests have a maximum of 100 points. Write your answer in interval notation.

360) To receive a B in a math course, an average between 80 and 89, inclusive, is required. Leah has scores of 76, 82 and 90 on her first three math tests. After the fourth test, she states that she now has a B average. In what range was her score on the fourth test for this statement to be true? Write your answer in interval notation.

361) To mail an envelope, Canada Post charges 37 cents for the first gram, and 25 cents for each additional gram. What is the weight of the envelope that can be mailed for $8.35? Write your answer in interval notation.

362) A car rents for $45 per day plus 21 ¢ per kilometer. You are at a daily budget of $66. How many kilometers can you cover to stay within your budget?

363) Anna is considering writing and publishing her own book. She estimates her revenue to be $R = 6.41x$ and her cost to be $C = 10039 + 1.09x$ where x is the number of books sold. Find the minimum number of books she must sell to make a profit.

364) Jan scored 65 points on the calculus midterm. If the final exam counts twice as much as the midterm exam, then what range of the final exam scores would result in the final average between 79 and 90? Both tests have a maximum of 100 points. Write your answer in interval form.

365) Canada post defines girth of a box as the sum of the twice the width and twice the width. The maximum length, height and width allowed are 2 m and length plus girth must be at most 3 m. If the box is 1 m long and 75 cm wide, in what range can the height be?

7.3 Linear Inequalities in Two Or More Variables

Graph by hand the solution region of each of the following systems of inequalities, identify its corner points and state whether the region is bounded or unbounded.

366) $x + y \leq 17$
$4x + y \geq 5$

367) $x + y < -2$
$4x + 2y \geq 5$

368) $2x - 3y > 10$
$4x + 3y > 5$

369) $x + 3y \leq 15$
$2x + y \leq 12$
$x \geq 0$
$y \geq 0$

370) $x + 2y \geq 50$
$3x + y \leq 90$
$x \geq 10$
$0 \leq y \leq 30$

371) A company produces a basic and premium version of its product. The basic version requires 20 minutes of assembly and 15 minutes of painting. The premium version requires 30 minutes of assembly and 30 minutes of painting. The company has staffing for 3,900 minutes of assembly and 3,300 minutes of painting each week. Represent graphically how many of each version can be produced under these conditions. Graph the feasible region and determine all of the corner points of the feasible region.

372) A health-food business would like to create a high-potassium blend of dried fruit in the form of a box of 10 fruit bars. It decides to use dried apricots, which have 407 mg of potassium per serving, and dried dates, which have 271 mg of potassium per serving. The company can purchase its fruit through in bulk for a reasonable price. The company would like the box of bars to have at least the recommended daily potassium intake of about 4700 mg, and contain at least 1 serving of each fruit. Represent graphically how many servings of each dried fruit should go into the box of bars under

these conditions. Graph the feasible region and determine all of the corner points of the feasible region.

373) Product A requires 3 hours of manufacturing and 1 hour of assembly. Product B requires 4 hours of manufacturing and 2 hours of assembly. There are a total of 84 hours of manufacturing and 32 hours of assembly available. Represent graphically how many of each product can be produced under these conditions. Graph the feasible region and determine all of the corner points of the feasible region.

374) One plant manufactures two types of product, each using two assembly lines. The first product requires two hours on the first assembly line and 4 hours on the second assembly line per unit. The second product requires four hours on the first assembly line and 2 hours on the second assembly line per unit. Both of the assembly lines are operational 24 hours per day. Represent graphically how many of each product can be produced over the course of 24 hours under these conditions. Graph the feasible region and determine all of the corner points of the feasible region.

375) A health facility must schedule its junior and senior staff to run its daily operations. At most 56 staff members are available per day and the facility needs at least 35 to run properly. They need at least 5 senior staff and can have at most 2 members of senior staff for every 5 members of junior staff. Cost of a junior staff member is $280 per day and $400 for a member of the senior staff. Represent graphically how many of the junior staff and how many of the senior staff they should schedule under these conditions. Graph the feasible region and determine all of the corner points of the feasible region.

7.4 Optimization Using Graphical Solutions

376) A company produces a basic and premium version of its product. The basic version requires 20 minutes of assembly and 15 minutes of painting. The premium version requires 30 minutes of assembly and 30 minutes of painting. If the company has staffing for 3,900 minutes of assembly and 3,300 minutes of painting each week. They sell the basic products for a profit of $30 and the premium products for a profit of $40. How many of each version should be produced to maximize profit?

377) A health-food business would like to create a high-potassium blend of dried fruit in the form of a box of 10 fruit bars. It decides to use dried apricots, which have 407 mg of potassium per serving, and dried dates, which have 271 mg of potassium per serving. The company can purchase its fruit through in bulk for a reasonable price. Dried apricots cost $9.99/lb. (about 3 servings) and dried dates cost $7.99/lb. (about 4 servings). The company would like the box of bars to have at least the recommended daily potassium intake of about 4700 mg, and contain at least 1 serving of each fruit. In order to minimize cost, how many servings of each dried fruit should go into the box of bars?

378) Product A requires 3 hours of manufacturing and 1 hour of assembly. Product B requires 4 hours of manufacturing and 2 hours of assembly. There are a total of 84 hours of manufacturing and 32 hours of assembly available. Determine the production to maximize profit if the profit on product A is $50 and the profit on product B is $60.

379) A plant manufactures two types of product, each using two assembly lines. The first product requires two hours on the first assembly line and 4 hours on the second assembly line per unit. The second product requires four hours on the first assembly line and 2 hours on the second assembly line per unit. Both of the assembly lines are

operational 24 hours per day. If the profit from the sales of the first product is $30 per unit and from the second $50 per unit, how many of each product should be produced over the course of 24 hours to maximize the profit?

380) A company has two employees at its disposal to produce at least 400 units of a product within 17 hours. One employee, paid at $18/h, can produce 20 units per hour while the other, paid at $21 per hour, can produce 25 units per hour. How many work-hours should be assigned to each of the two employees to minimize the work compensation cost?

381) A toy manufacturer makes dolls, with Barb and Kin being its two top sellers. It takes 5 units of raw material and 3 units of time to assemble a Barb doll. It takes 3 units of raw material and 5 unit of time to assemble a Kin doll. On a given day the manufacturer has at most 150 units of raw material and 180 units of time. If the manufacturer makes a profit of $12 on each Barb and $10 on each Kin, how many of each doll should they make in order to maximize the profit?

382) A school district office has reached out to transportation operators in the area for price quotes on renting busses for a class trip. One of the operators has two types of vehicles at its disposal: a regular bus and a mini-bus. Each regular bus can carry 48 students in addition to 5 required chaperons, and each mini-bus can carry 8 students in addition to 1 chaperone. To achieve desired profit margins, the operator must rent a regular bus for $1,100 and a mini-bus for $100. If there are 624 students in the class that are eligible to go on the trip and only 70 parents have volunteered to serve as chaperones, how many vehicles of each type should the operator include in their quote to the school district office in order to make it most competitive? What is the minimal quote they can put forward?

383) An investor has $60,000 to invest in a stock and a mutual fund. The stock yields 8% per year and the mutual fund yields 5%. The mutual fund requires a minimum investment of $9,000, and the investor requires that at least twice as much should be invested in stock as in the mutual fund. How much should be invested in the stock and how much in the mutual fund to maximize the return? What is the maximum return?

384) An auto glass repair chain plans to expand by opening several new repair shops. The chain operates two types of car repair shops: Quick-Fix and Full Service. A Quick-Fix shop costs $100,000 to build, requires 5 employees and has an expected annual revenue of $200,000. A Full-Service shop costs $150,000 to build, requires 15 employees and has an expected annual revenue of $500,000. The chain has $2,400,000 available for construction in the expansion budget. Labour contracts require that they hire no more than 200 employees, and licencing restrictions require that they open no more than 20 new car repair shops. How many repair shops of each type should the chain open in order to maximize the expected revenue? What is the maximum expected revenue? How much of their construction budget will they use and how many employees will they hire?

385) A health facility must schedule its junior and senior staff to run its daily operations. At most 56 staff members are available per day and the facility needs at least 35 to run properly. They need at least 5 senior staff and can have at most 2 members of senior staff for every 5 members of junior staff. Cost of a junior staff member is $280 per day and $400 for a member of the senior staff. How many of the junior staff and how many of the senior staff should they schedule to minimize daily staffing costs? What would be the daily staffing costs in that case?

7.5 Optimization using Excel Solver

For each of the following:

- Set up the problem
    - Define the variables
    - State the objective and determine the objective function
    - List and describe the constraints
- Use Excel Solver to determine a solution, if any

386) A city has at its disposal three potential suppliers for obtaining 10 custom-made accessibility ramps on a short notice. The city budgeted $7600 for the order. The first supplier charges $700 per ramp, the second $400 and the third $800. Because of the short notice, the first supplier can commit to a maximum of 6 ramps, the second supplier to maximum 3 ramps, and the third supplier to maximum 8 ramps. If it takes 3 labour-days to install the first supplier's ramp, 5 labour-days to install the second supplier's ramp and 1 labour-day to install the third supplier's ramp, how many ramps should the city order from each supplier in order to minimize the labour costs for installation of all ten ramps?

387) A product is being made in three factories, Factory 1 in Oshawa, Factory 2 in Windsor, and Factory 3 in Cornwall. Each factory produces at least 600, 800 and 1000 units respectively. In support of the just-in-time delivery system, the product then has to get distributed to the three warehouses, A, B and C. Capacity of each warehouse is 700, 1200 and 500. The cost of delivery from each factory to each warehouse is listed in table below. How many units of product should be shipped from each factory to each warehouse in order to minimize the cost? What is the minimal cost?

From/To	Cost of Shipping		
	Warehouse A	Warehouse B	Warehouse C
Factory 1	$ 6	$ 5	$ 3
Factory 2	$ 8	$ 10	$ 8
Factory 3	$ 11	$ 14	$ 18

388) Sky's the Limit specializes in making model airplanes, including the following models: Vought, Avro, Blackbird, and Triplane. The production requires use of three machines – a metal casting machine, a paint spray machine, and a packaging machine. The time, in hours, each machine works to make each type of model, maximum hours available, and the profit for each model are given in table below. Determine how many of each model airplanes the company should produce to maximize the profit.

	Vought	Avro	Blackbird	Triplane	Maximum Time
Metal Casting	2.1 h	2.5 h	1.9 h	1.5 h	4000 h
Paint Spray	0.9 h	0.5 h	0.5 h	0.8 h	1600 h
Packaging	0.5 h	0.5 h	0.6 h	0.5 h	1000 h
Profit	$ 1.50	$ 1.60	$ 1.60	$ 1.30	

389) A farmer has at most 400 acres of land suitable to cultivate corn, sugar beets, turnips and potatoes. Corn requires 25 hours of labour per acre, sugar beets 30 hours, turnips 35, and potatoes require 40 hours of labour per acre, but there is a maximum of 5800 hours of labour available. The cost of cultivating corn, sugar beets, turnips and potatoes is $15, $35, $40 and $20 per acre, respectively. The farmer has a maximum of $21,000 available for land cultivation. If the farmer expects to make a profit of $65, $55, $50 and $35 per acre for corn, sugar beets, turnips, and potatoes respectively, how many acres of each crop should the farmer cultivate to maximize profit?

390) The MountainFood Co-Op operating at Banff National Park sells granola snack bags to hikers and other visitors. They source their ingredients from blends provided by local food suppliers, Momma's Kitchen, Best Fud, and Gotta Eat. Each supplier's blend contains different amount of four key granola ingredients: oats, raisins, almonds and chocolate chips, as follows:

Ingredient	Momma's Kitchen Blend	Best Fud Blend	Gotta Eat Blend
oats	30 g	20 g	50 g
raisins	20 g	30 g	10 g
almonds	40 g	0 g	20 g
chocolate chips	60 g	80 g	50 g

The minimum amount of oats, raisins, almonds and chocolate chips per kg of granola are 640 g, 800 g, 160 g and 1280 g respectively (long live chocolate!). The cost per kg of Momma's Kitchen blend is $3, Best Fud's $4, and Gotta Eat's $2. Determine how many kg of each blend MountainFood Co-Op should purchase from each supplier in order to minimize cost and state the cost.

391) Distribution Center A_1 has 20 units of a certain product and Distribution Center A_2 has 40 units of the same product. This product has to be moved to stores B_1, B_2 and B_3 in the following way: 8 units must be moved to B_1, 28 units to B_2, 14 units to B_3, and 15 units to B_4. Organize the transportation in such a way that the cost of transportation is minimized, given the following cost per unit associated with transporting the product from the distribution centers to the stores:

Cost per unit from A_i to B_j	B_1	B_2	B_3	B_4
A_1	$4	$9	$3	$2
A_2	$4	$8	$1	$3

392) A company manufactures two products, P_1 and P_2, out of four types of raw materials: m_1, m_2, m_3, and m_4. Each of the two products requires a certain amount of each of the materials, whose supplies are limited. The amount of each material required for each of the products, as well as the profit gained from sales of each of the product are outlined in the table below. Find the optimal number of products to maximize profit.

Type of material	Available supply of material	Product type	
		P_1	P_2
m_1	19	2	3
m_2	13	2	1
m_3	15	0	3
m_4	18	3	0
	Profit	$7/unit	$5/unit

393) A health facility must schedule its junior and senior staff to run its daily operations. At most 56 staff members are available per day and the facility needs at least 35 to run properly. They need at least 5 senior staff and can have at most 2 members of senior staff for every 5 members of junior staff. Cost of a junior staff member is $280 per day and $400 for a member of the senior staff. How many of the junior staff and how many of the senior staff should they schedule to minimize daily staffing costs? What would be the daily staffing costs in that case?

394) A catering company is to make lunch for a business meeting. It will serve ham sandwiches, light ham sandwiches, and vegetarian sandwiches. A ham sandwich has 1 serving of vegetables, 4 slices of ham, 1 slice of cheese, and 2 slices of bread. A light ham sandwich has 2 serving of vegetables, 2 slices of ham, 1 slice of cheese and 2 slices of bread. A vegetarian sandwich has 3 servings of vegetables, 2 slices of cheese, and 2 slices of bread. A total of 10 bags of ham are available, each of which has 40 slices; 18 loaves of bread are available, each with 14 slices; 200 servings of vegetables are available, and 15 bags of cheese, each with 60 slices, are available. Given the resources, how many of each sandwich can be produced if the goal is to maximize the number of sandwiches?

395) A factory manufactures three products, A, B, and C. Each product requires the use of two machines, Machine I and Machine II. The total hours available, respectively, on Machine I and Machine II per month are 180 and 300. The time requirements and profit per unit for each product are listed below.

	A	B	C
Machine I	1	2	2
Machine II	2	2	4
Profit	20	30	40

How many units of each product should be manufactured to maximize profit, and what is the maximum profit?

396) For optimal health and the ability to work, over the course of 24 hours a person needs a certain amount of nutrients; for example: 10 units of fat, 12 units of protein, 16 units of carbohydrates, 10 units of water and 1 unit of vitamins. Suppose food items A and B contain these ingredients as listed in the table below. If A costs $2 and B costs $3 per unit, how many units of each should be consumed in order to minimize the cost while satisfying the daily nutritional requirement?

Amount of nutrients per unit of product	Fat	Protein	Carbohydrates	Water	Vitamins
A	1	3	2	2	1
B	5	2	4	2	0

397) A company is creating a meal replacement bar. They plan to incorporate peanut butter, oats, and dried cranberries as the primary ingredients. The nutritional content of 10 grams of each is listed below, along with the cost, in cents, of each ingredient. Find the amount of each ingredient they should use to minimize the cost of producing a bar containing a minimum of 15g of each ingredient, at least 10g of protein and at most 14g of fat.

	Peanut Butter, 10g	Oats, 10g	Cranberries, 10g
Protein (grams)	2.5	1.7	0
Fat (grams)	5	0.7	0.1
Cost (cents)	6	1	2

398) A factory manufactures three products, A, B, and C. Each product requires a certain number of hours of manufacturing, assembly and finishing time, shown below, along with the total time available and profit. Find production levels to maximize profit.

	A	B	C	Total Hours Available
Manufacturing	4	5	2	300
Assembly	7	4	1	250
Finishing	4	5	1	200
Profit	175	130	40	

399) A factory manufactures chairs and tables, each requiring the use of three operations: Cutting, Assembly, and Finishing. The first operation can be used at most 40 hours; the second at most 42 hours; and the third at most 25 hours. A chair requires 1 hour of cutting, 2 hours of assembly, and 1 hour of finishing; a table needs 2 hours of cutting, 1 hour of assembly, and 1 hour of finishing. If the profit is $20 per unit for a chair and $30 for a table, how many units of each should be manufactured to maximize profit?

400) A distribution company needs to ship products from its two warehouses to three retailers. Warehouse A has 1000 products in stock, and Warehouse B has 1200 products. Retailer 1 needs 700 products, Retailer 2 needs 500 products, and Retailer 3 needs 600 products. The cost to ship a product from each warehouse to each retailer is shown below. Find the number of products the company should ship from each warehouse to each retailer to minimize shipping costs.

	Retailer 1	Retailer 2	Retailer 3
Warehouse A	3	5	6
Warehouse B	4	7	5

Chapter 8 Functions as Relationships

8.1 Functions

401) At a coffee shop, the menu consists of items and their prices. Is price a function of the item? Is the item a function of the price? Justify your answers.

402) At Ontario Tech University, the final grade a student earns in the course corresponds to a letter grade (see Ontario Tech - Grading). Is the letter grade a function of the percentage a student can obtain in the course? Is the percentage a student can obtain in the course a function of the letter grade? Justify.

403) Consider information related to bank accounts.

   a) Is the account balance a function of the bank account number? Justify.

   b) Is the bank account number a function of the account balance? Justify.

404) Introduce function notation of your choice using variable names to represent a function that takes as input the name of a month, and gives as output the number of days in that month. State clearly what each variable represents.

405) A function $N(y)$ gives the number of police officers, N, in a town in year y. What does $N(2005) = 300$ tell us?

406) Which of these tables define a function (if any)? Justify.

a)

Input	Output
2	1
5	3
8	6

b)

Input	Output
-3	5
0	1
4	5

c)

Input	Output
1	0
5	2
5	4

407) Using the table shown for $Q(n)$:

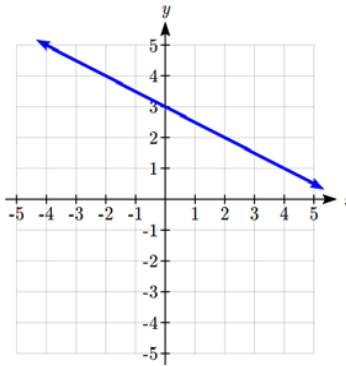| n	1	2	3	4	5
Q	8	6	7	6	8

   a) Evaluate $Q(3)$ and interpret your result (describe in words what the answer means).

   b) Solve $Q(n) = 6$ and interpret your result.

   c) Evaluate $Q(7)$. Interpret your result.

   d) Solve $Q(n) = 1$. Interpret your result.

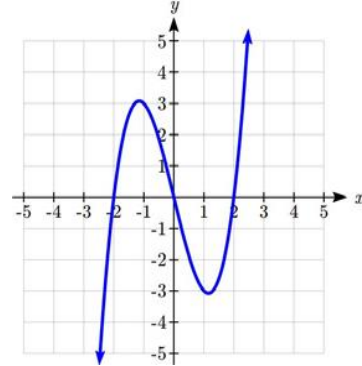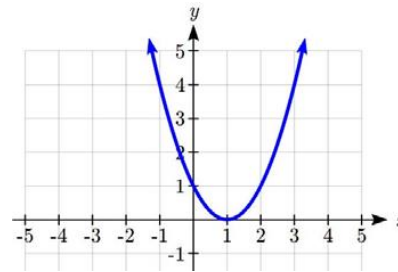**408)** Which of these graphs defines a function? Justify your answers.

a)
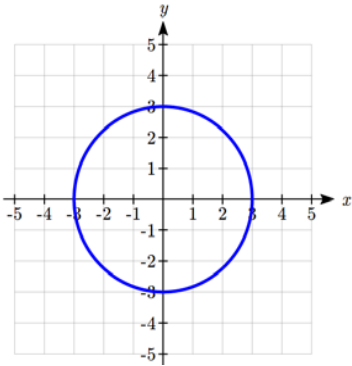
b)

c)

409) Given the graph shown on the right,

   a) Evaluate $f(2)$.

   b) Solve $f(x) = 4$.

   c) Evaluate $f(-1)$.

   d) Solve $f(x) = 1$.

410) If possible, express the relationship $2n + 6p = 12$ as a function $p(n)$.

411) Consider the relationship $3m^2 - 15n = 7$. Use this relationship to express the following functions, if possible. If this is not possible, justify why not.

   a) Express n as a function of m.

   b) Express m as a function of n.

412) Given the function $k(t) = t^3 + 2$.

   a) Evaluate $k(2)$.

   b) Solve $k(t) = 1$.

413) Given the function $h(p) = p^2 + 2p$,

   a) Evaluate $h(4)$.

   b) Solve $h(p) = 3$.

414) Given the function $g(m) = \sqrt{m - 4}$

   a) Evaluate $g(5)$.

   b) Solve $g(m) = 2$.

415) A data storage company rents server space for a flat annual fee of $150 and storage charge of $0.39 per GB for less than 1.8 petabytes per annual contract (one petabyte is 1000 terabytes and one terabyte is 1000 GB).

   a) Write the formula for the total cost C to rent server space as a function of the gigabytes of data storage requirement s.

   b) What is the domain of this function?

   c) What is the range of this function?

   d) Find the total cost to rent server space for 3.5 terabytes of storage space.

   e) Determine how much server space was rented under the annual contract if the bill was $1203.

416) A rental car company rents cars for a flat fee of $20 and an hourly charge of $10.25. Reservations made but cancelled are charged the flat fee. The company policy states that the rentals must be less than 5 days in duration.

   a) Write the formula for the total cost C to rent a car as a function of the hours h the car is rented.

   b) What is the domain of this function?

   c) What is the range of this function?

   d) Find the total cost to rent a car for 2 days and 7 hours.

   e) Determine how long the car was rented if the bill is $481.25.

417) In making strategic purchasing decisions in business, we study the relationship between the market supply of an item and the price of the item charged by the suppliers. Suppose that for a particular product the number x of units available and the price p per unit demanded by the suppliers satisfy the following relationship:

$$-2x + 35p = 7000$$

   a) If possible, express the supply as a function of the price and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.

   b) If possible, express the price as a function of the supply and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.

418) In making strategic decisions about product pricing, we study the relationship between market demand for an item and the price of the item. Suppose the market demand for x number of units and the price of the unit p of a particular product satisfy the following relationship:

$$9x + 150p = 6500$$

   a) If possible, express the demand as a function of the price and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.

   b) If possible, express the price as a function of the demand and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.

419) If the revenues R from sales of x units of a certain product and the cost C of manufacturing x units of the same product can be expressed as

$$R(x) = 18x \text{ and } C(x) = 9.37x + 25000$$

a) Determine the profit function P in terms of the number of units x.

b) Calculate $P(2000)$ and interpret what that means.

c) Calculate $P(4000)$ and interpret what that means.

d) How many units would they have to produce and sell to break even?

e) How many units would they have to produce and sell to earn $20,000 in profit?

420) The population of Oshawa in the year 1960 was 77,000 people. Since then the population has grown to 379,848 people reported during the 2016 census. Choose descriptive variables for your input and output and use interval notation to write the domain and range.

421) Describe the domains of the following functions using interval notation:

a) $f(x) = \frac{3x+5}{x^2+4x-5}$

b) $f(x) = \frac{\sqrt{2x-3}}{\sqrt{5-x}}$

c) $f(x) = \frac{x}{\sqrt[3]{x+3}}$

422) A phone data plan has a basic charge of $30 a month. The plan includes first 2GB free and charges $10 for each additional GB, up to 8 GB total usage, after which it charges $15 for each additional GB. If d is the amount of data used (in GB) and C is the total monthly cost:

a) Express $C(d)$ as a (piece-wise) formula.

b) Identify the independent and the dependent variables of C.

c) Identify the domain and the range of C.

d) Graph C as a function of d for $0 \le d \le 10$.

e) Calculate the cost if 9GB were used.

423) Describe the relationship shown in the graph below as a function and describe its domain and range in interval form.

Estimated monthly light crude oil production (Alberta)

Source: open.canada.ca
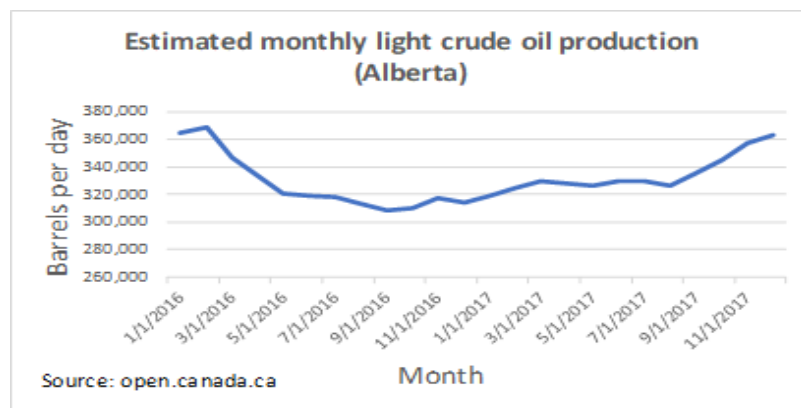
424) An analysis of past marketing campaigns focused on the relationship between the weekly campaign expenses E ($) and the quantity of the product sold q (in thousands) each week. Using the data collected, the analysts created the following model:

$$q(E) = 1015.71 - 3.51071E + 0.00592857E^2 - 0.0000025E^3 \quad \text{where} \quad 600 \leq E \leq 1000$$

a) What is the independent variable in this relationship? State its description, notation, and unit of measurement.

b) What is the dependent variable in this relationship? State its description, notation, and unit of measurement.

c) In the graph of this function shown to the right, what does the horizontal axis represent?

d) What does the vertical axis represent?

e) State the domain of the function.

f) State the range of the function.

g) Describe the meaning of the point $(750, 662.811)$ shown on the graph.

h) Write the point $(750, 662.811)$ using the function notation.
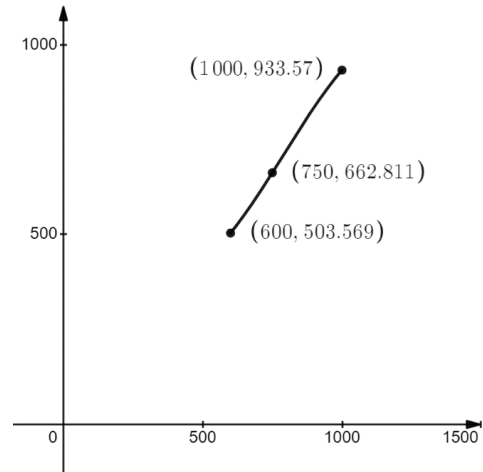
425) An analysis of the impact of computer architecture on a data center network focused on the relationship between average task completion time t (ms) and the required link bandwidth b (Gbps). Using the data collected, the analysts created the following model:

$$b(t) = \frac{0.58t - 4.872}{t^2 - 6.4t + 2.4} \quad \text{where} \quad 0.99 \leq t \leq 1.97$$

a) What is the independent variable in this relationship? State its description, notation, and unit of measurement.

b) What is the dependent variable in this relationship? State its description, notation, and unit of measurement.

c) Briefly explain how you determined what the independent variable is and what the dependent variable is.

d) In the graph of this function shown to the right, what does the horizontal axis represent? Justify.

e) What does the vertical axis represent? Justify.

f) State the domain of the function.

g) State the range of the function.

h) Calculate $b(1.5)$, rounding your answer to three decimals, plot it on the given graph, including the coordinates, and briefly explain what your result means in the context of this question.
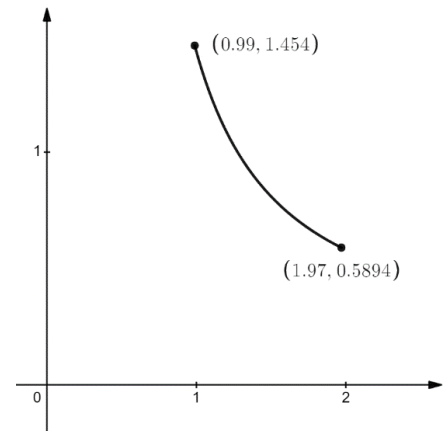
8.2 Linear Functions

426) Sketch the graph of the following function.

$$f(x) = \frac{2}{3}x + 1$$

427) Find the x- and y-intercepts of the following lines and graph the lines:

a) $\frac{x}{2} - \frac{y}{3} = 600$

b) $0.09x - 0.06y = 54$

428) Find the slope of the following lines and graph the lines:

a) $3x - 5y = 1.5$

b) $y + 11 = 0$

429) Find the equations of the lines passing through the following points, identify x- and y-intercepts and slope for each, and sketch their graphs.

a) $(-2, 0)$ and $(0, 3)$

b) $\left(\frac{3}{4}, -3\right)$ and $\left(-5, \frac{1}{8}\right)$

430) Consider the points (3, 2) and (5, -2) and the linear function whose graph passes through them.

a) Find the slope of the line.

b) Find the slope-intercept form of the equation of the line.

c) Write the equation of the line in the standard form.

431) Sarah wants to go skating at Super Skate ice rink. She has to pay a $7 entrance fee and $1.25 for every minute she is on the rink.

a) Write an equation to determine the cost (C) in terms of the number of minutes (t) that she is on the rink.

b) If she only has $43.25, find the number of minutes she can be on the rink.

432) If you earn $30,000 per year and you spend $29,000 per year, write amount of money you save A after y years, assuming you start with no money.

433) Given the two points (2, 3) and (0, 4), find the rate of change. Is this function increasing or decreasing?

434) The balance in your college payment account C, is a function of the number of quarters q, you attend. Interpret the function $C(q) = 20000 - 4000q$ in words and explain the meaning of each number and symbol in this equation. How many quarters of college can you pay for until this account is empty?

435) Graph $f(x) = 5 - \frac{2}{3}x$ using the vertical intercept and slope. Using the fact that the slope $-2/3$ could also be written as $\frac{2}{-3}$, find a point on the graph that has a negative x value.

436) Consider the graph of $j(t) = 5 - t$ and determine the following for the function $j(t)$:

    a) Vertical intercept coordinates

    b) Horizontal intercepts coordinates

    c) Slope

    d) Is j(t) an increasing or decreasing function (or neither)

437) A company purchased $120,000 in new office equipment. Then expect the value to depreciate (decrease) by $16,000 per year. Find a linear model for the value, then find and interpret the horizontal intercept and determine a reasonable domain and range for this function.

438) A manager for a country market will spend a total of $80 on apples at $0.25 each and pears at $0.50 each. Write the number of apples she can buy as a linear function of the number of pears. Find the slope and interpret your answer. Graph the function.

439) At a price of $2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of $2.37 per bushel, the supply is 7,900 million bushels and the demand is 7,800 bushels.

    a) Assuming that price and supply are linearly related, determine the price in terms of supply (the *price-supply equation*).

    b) Assuming that price and demand are linearly related, determine the price in terms of demand (the *price-demand equation*).

    c) Find the equilibrium point (price and the number of units for which supply and demand are equal).

    d) Graph the price-supply equation, price-demand equation and the equilibrium point in the same coordinate system.

440) A plant can manufacture 50 tennis racquets per day for a total daily cost of $3,855 and 60 tennis racquets per day for a total daily cost of $4,245.

    a) Assuming that daily cost and production are linearly related, find the total daily cost C of producing x tennis racquets.

    b) Interpret the slope and y-intercept of this cost equation.

    c) Graph the total daily cost for $0 \leq x \leq 100$.

441) NewTech Wireless company offers a monthly calling plan where the total cost is linearly related to the number of minutes used. Given that the total monthly cost for 100 minutes used is $35.00 and that for 200 minutes the cost is $45.00:

    a) Express the cost C in terms of the number of minutes used t.

    b) What is the domain and the range of this function?

    c) What is the basic cost for the plan and what is the cost per minute?

    d) Sketch the graph of this function.

    e) What will be the cost if 400 minutes are used in a month?

    f) If the total cost for a month was $40.00, how many minutes were used?

442) A security company purchases a new security van for $53,000 and assumes that in 5 years it will have a trade-in value of $28,000.

    a) Find the linear model for the depreciated value V of the van after t years.

    b) What is the depreciated value of the van after 3 years?

    c) When will the depreciated value fall below $23,000?

    d) Interpret the slope and the y-intercept of V(t) (explain what the slope and the y-intercept represent in this context).

443) At $10 per ticket, Willie Williams and the Wranglers will fill all 8,000 seats in the Assembly Center. The manager knows that for every $1 increase in the price, 500 tickets will go unsold.

    a) Write the number of tickets sold n as a function of the ticket price p.

    b) What are the limits of the independent variable, if any?

444) The manufacturer of a new type of frying pan has calculated the monthly fixed costs to be $83,000 and variable costs of $7.35 for each frying pan produced. The pans are sold to a distributor for $20 per pan. The monthly manufacturing capacity is 20,000 units.

    a) Write the monthly cost function C in terms of number of units produced x. Determine its domain and range and graph it.

    b) Write the monthly revenue function R in terms of number of units produced x. Determine its domain and range and graph it.

    c) Write the monthly profit function P in terms of number of units produced x, assuming that all units produced are sold. Determine its domain and range and graph it.

    d) What will be their profit/loss if they are running at 25% capacity? At 75% capacity?

    e) How many units must they produce and sell in a month to break even? What percent is that of production capacity? What must be the sales to break even?

    f) How many units must they produce and sell in a month to make $100,000 in profit?

445) A manufacturing company, under contract to deliver a new line of beer bottles, estimates that it would cost $41,000 to produce 80,000 bottles while it would cost $59,000 to produce 120,000 bottles. Based on the cost analysis of previous production runs, they determined that that a linear model would best represent the costs of production.

    a) Assuming that the cost C and the number of bottles produced x are linearly related, determine the cost function $C(x)$.

    b) What are the fixed costs in this model and what is the variable cost per bottle?

8.3 Quadratic Functions

446)   Solve the following equations.

   a) $16x^2 - 24x + 9 = 0$

   b) $x^2 = 6x - 13$

   c) $1.5x^2 - 6.3x - 10.1 = 0$

447)   For each of the functions below, determine the domain and range of the function, the minimum or maximum value and where it occurs.

   a) $f(x) = 2x^2 + 8$

   b) $g(x) = -500x^2 + 3000x - 5000$

448)   Solve the following equations for the given variable:

   a)   Solve $5m^2 + 35m - 40 = 0$

   b)   Find the roots of $(w - 1)(w - 2) = 6$

   c)   Solve $7 + \frac{8}{p} = \frac{12}{p^2}$

   d)   Find all real solutions to $\frac{x-12}{3-x} = \frac{x+16}{x+1}$

449)   A backyard farmer wants to enclose a rectangular space for a new garden. She has purchased 80 feet of wire fencing to enclose 3 sides, and will put the 4th side against the backyard fence. Find a formula for the area enclosed by the fence if the sides of fencing perpendicular to the existing fence have length L. What dimensions should she make her garden to maximize the enclosed area?

450)   A local newspaper currently has 84,000 subscribers, at a quarterly charge of $30. Market research has suggested that if they raised the price to $32, they would lose 5,000 subscribers. Assuming that subscriptions are linearly related to the price, what price should the newspaper charge for a quarterly subscription to maximize their revenue?

451)   A company is planning to sell a new smart fitness device. Developing the product will cost $700,000, and each product will cost $30 to manufacture. Market research suggests that if they sell the device for $100, they will be able to sell 30,000 items. For each $10 they lower the price, they estimate they will sell 5,000 more items. Assuming quantity demanded is linearly related to price, determine the price that will maximize profit.

452)   The supply for a certain product can be modeled by $p = 3q^2$ and the demand can be modeled by $p = 1620 - 2q^2$, where p is the price in dollars, and q is the quantity in thousands of items. Find the equilibrium price and quantity.

453)   A company has determined the price-demand function for one of its products to be

   a) $p = 10 - 0.001x \qquad 1 \le x \le 10,000$

   b)   Write the company's revenue R in terms of number of units x.

   c)   If the cost can be calculated using $C = 7,000 + 2x$, find the break-even point(s).

   d)   Determine the profit P in terms of x and find the number of units to be produced and sold in order to make a profit of $5,000.

454) A company keeps records of the price from the sale of x units (in thousands) of a product. It determines that the price-demand function is given by $p(x) = 300 - x$. It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x) = 40x + 1600$.

a) Determine the revenue function and identify its domain.

b) Sketch the graph of the revenue function.

c) When will the company reach maximum revenue? What is the maximum revenue?

d) Find the break-even points for this company. (Round answers to nearest 1000.)

e) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

455) The price-demand equation for a certain product is

$$p = 50 - 0.002x$$

where x is the number of units sold per week and p is the price in dollars at which each one unit is sold. The weekly revenue is given by $R = xp$. What number of units sold produces a weekly revenue of $24,500? What is the price they should charge to achieve that revenue level?

456) Suppose that a small Oshawa pottery factory has a daily fixed cost of $3,600 and can manufacture 75 specialty tea sets per day for a total daily cost of $6,000. Suppose its revenue function in terms of number of tea sets x sold per day can be modelled by

$$R(x) = -x^2 + 250x \text{ for } 0 \le x \le 250$$

a) Determine the cost function assuming the production and the cost are linearly related.

b) Determine the average cost function $\bar{C}(x)$.

c) What does the average cost tend to as the production increases and why does the answer make sense in the context of production costs?

d) Find the profit function and state its domain.

e) Find the break-even point(s) and the maximum profit.

f) Sketch the graph of the profit function.

457) A game developer plans to put a new game on the market. They estimate that they would be able to sell 500,000 games if they price the game at $60 and 750,000 if they price the game at $45. If they assume that the relationship between price and demand is linear:

a) Express price p in terms of demand x and explain why this may be useful to the developer.

b) Find the domain of p and interpret what that means.

c) What price should they charge if they wish to sell 900,000 games?

d) What can they expect the demand to be if they set the price at $70?

e) Express demand x in terms of price p and explain why this may be useful to the developer.

458) Suppose that a plant has a daily fixed cost of $4,000 and can manufacture 50 guitars per day for a total daily cost of $6,000. Suppose its revenue function can be modelled by $R(x) = -x^2 + 200x$, where x is the number of guitars sold.

   a) Determine the cost function.

   b) Find the profit function.

   c) Find the break-even point(s).

459) A marketing consultancy is planning to sell a new app for small business that would help the businesses tailor their marketing plans. Developing the app will cost $450,000, and each app subscription will cost $150 per year to support. Market research suggests that if they sell the app subscriptions for $1000 per year, they will be able to sell 2,000 annual subscriptions. For each $100 reduction in price, they estimate they will sell 500 more annual app subscriptions. Assuming the quantity demanded is linearly related to price:

   a) Analyze the revenue:

      i. Determine the revenue function in terms of demand and its domain.

      ii. Find the number of app subscriptions and the price per subscription that will produce the maximum and the minimum revenue and state the what the maximum and minimum revenue are, if any.

   b) Analyze the cost:

      i. Determine the cost function in terms of units produced and its domain.

      ii. Determine when the cost will be at its maximum and at its minimum and state what the maximum and minimum costs are, if any.

   c) Analyze the profit:

      i. Determine the profit function in terms of units produced and sold and its domain.

      ii. State when the profit will be at its maximum and what the maximum profit is, if any.

      iii. Determine the number of annual app subscriptions the company would have to sell and the price they would have to charge to break even.

   d) Graph the cost-volume-profit chart (revenue, cost and profit on the same graph) and identify on the chart the break-even points and the areas of profit and loss.

460) An appliance company's marketing research department established that, for their new model of a food processor, the price-demand function is

$$p(x) = -0.05x + 114$$

where p is the price in dollars and x is the demand in thousands of food processors.

   a) Determine the domain of p.

   b) Find the revenue function in terms of demand and its domain.

   c) Sketch the graph of the revenue function.

   d) Find the number of units sold that will produce maximum revenue and determine the maximum revenue.