

## Section 8.6 Logarithmic Functions

- 1) Solve the following exponential equations for a given variable using natural logarithm  $\ln$ . Round your answers, where appropriate, to four decimals.

a)  $5 = 3^t$

b)  $2^x = 0.3$

c)  $4 = (1.04)^m$

d)  $40 = 10 \cdot 6^t$

e)  $3 \cdot 15^n = 25$

f)  $e^{3k} = 5000$

g)  $0.4 - 5e^{-2t} = 1.5$

h)  $4e^{-0.02t} = 8.2e^{-0.03t}$

i)  $3P = Pe^{0.0004t}$ ; solve for  $t$

j)  $3P = P \left(1 + \frac{r}{m}\right)^{mt}$ ; solve for  $t$

k)  $A = P(1 + i)^n$ ; solve for  $n$

l)  $A = P \left(1 + \frac{r}{m}\right)^{mt}$ ; solve for  $t$

m)  $A = Pe^{rt}$ ; solve for  $t$

n)  $A = Pe^{rt}$ ; solve for  $r$

- 2) Assuming the graphs below represent logarithmic functions, match the graphs with their corresponding function from the list below:

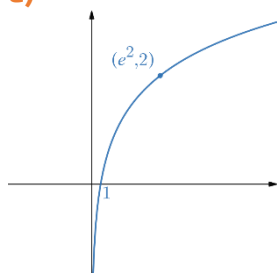
A.  $f(x) = \ln x$

B.  $f(x) = 15 \log_5 x$

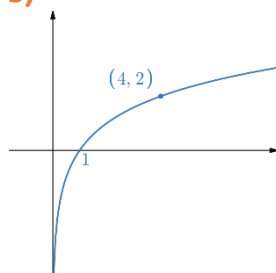
C.  $f(x) = \log_2 x$

D.  $f(x) = -0.5 \ln x$

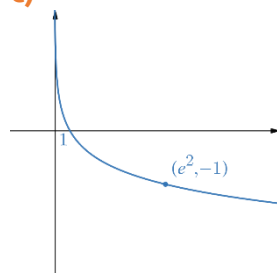
a)



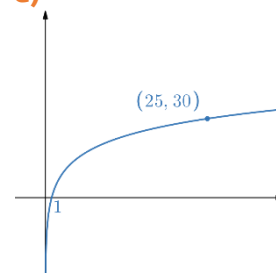
b)



c)



d)



- 3) The following functions are used to project the population size (in thousands) of three municipalities in  $t$  years from now.

A.  $P(t) = 625(1.06)^t$

B.  $P(t) = 43(1.13)^t$

C.  $P(t) = 1187(0.98)^t$

D.  $P(t) = 12(0.87)^t$

- a) For the municipalities whose population is increasing, determine when the population is projected to double.

- b) For the municipalities whose population is decreasing, determine when the current population is projected to be 30% less than now.

- 4) The Consumer Price Index (CPI) is used as an indicator for inflation, or the change in consumer's purchasing power over time. Bank of Canada's monetary policies are aimed at keeping the inflation close to 2%. If in 2002 the CPI was valued at 100, write the formula for the CPI function  $I(t)$ , where  $t$  is years since 2002, which has CPI increasing by 2% per year. Using this model, when would you expect the CPI to be 131.95?
- 5) The data on global revenues from consumer electronic sales shows 3.5% year-over-year increase. If the global consumer electronic revenues were \$850 million in 2012, determine the function modeling the global revenues (in millions of dollars)  $t$  years from 2012 and estimate when the global revenue would reach \$1.6 trillion.

- 6) A soil decontamination company bidding on a contract with a municipality to clean up an abandoned industrial site estimates that the percentage of soil contamination will decrease by 15% with each \$1000 increase in cost.
- a) Write the function  $p(x)$  that describes the percentage of soil contamination (measured in %) at the cost of  $x$  thousand dollars if the current, pre-cleanup, contamination is estimated to be 65% and determine what the cost would be to reach the soil contamination level of 10%.
  - b) Use the formula you determined for  $p(x)$  in a) to write the cost  $x(p)$  (in thousands of dollars) as a function of the percentage of soil contamination  $p$ . Use  $x(p)$  to determine what the cost would be to reach the soil contamination level of 10%.
- 7) An insurance broker started a job with an insurance company with 10 clients. The company's expectation from its brokers is to maintain a monthly increase by 5% percent in the number of new clients. Determine the function  $n(t)$  that describes the number of clients the broker should have after  $t$  months. Use this function to write the number of months  $t(n)$  needed to sign on  $n$  new clients. How many months does the broker have to double their initial number of clients?
- 8) For each of the following, write the function  $A(t)$  that describes the return on an investment of \$2300 in  $t$  years with interest rate of 1.5% and determine how long it will take to increase the initial investment by 20% if the interest is:
- a) compounded monthly
  - b) compounded continuously
- 9) For each of the following, determine the function  $I(t)$  that describes the interest in  $t$  years owed on a loan in the amount of \$23,000 with interest rate of 2.57% and determine how long it will take for interest amount to grow to \$2,400 if the interest is
- a) compounded quarterly
  - b) compounded continuously
- 10) Suppose \$5000 is invested in an account earning interest at 4.5% annual rate. Determine how long it will take to double the investment if the interest is compounded
- a) annually
  - b) continuously
- 11) You want to invest \$5,000 you received from family and friends upon the birth of your child into a Registered Education Savings Plan using a Guaranteed Income Certificate (GIC). You have two options: monthly compounding at 2.156% or continuous compounding at 2.155%. How many years will it take with each option to earn \$500 in interest on your investment?
- 12) A local restaurant estimates that weekly sales  $s$  and weekly advertising costs  $x$  (both in dollars) are related by

$$s(x) = 14,000 - 13,000e^{-0.0006x}$$

- a) Determine  $x(s)$  and briefly explain why this would be useful to the restaurant managers.
- b) How much should they invest in advertising to achieve \$13,000 in weekly sales?