Section 2.8 Curve Sketching

1) Consider the graphs of the functions below and answer the following questions.

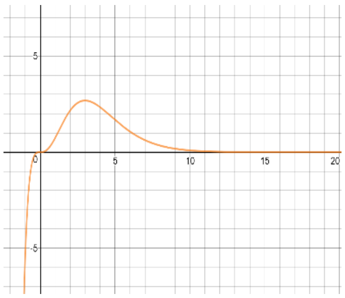
a) What is the behaviour of the function on the interval shown?

b) What are the "points of importance"?

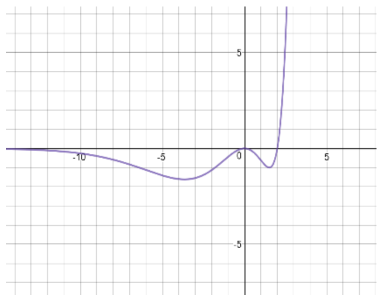
c) What can you say about the rate of change of the function throughout the interval shown?

d) Draw a rough sketch of the graph for the derivative of the function.

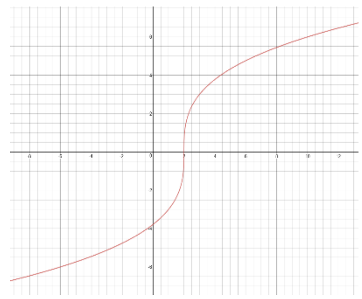
Graph A



Graph B



Graph C



2) Complete the following statements, if possible.

a) When the function is increasing, the derivative is ___________.

b) When the derivative is positive, the function is ___________.

c) When the function is decreasing, the derivative is ___________.

d) When the derivative is negative, the function is ___________.

e) When the function is positive, the derivative is ___________.

f) When the function is negative, the derivative is ___________.

3) Consider the table provided with information about the function $f(x)$ and complete the questions below.

x	$(-\infty, 0)$	0	$(0,3)$	3	$(3,5)$	5	$(5,8)$	8	$(8,\infty)$
$f(x)$	positive	2	positive	0	negative	-4	negative	-2	negative
$f'(x)$	positive	0	negative	negative	negative	0	positive	0	negative

a) The zero(s) of f is (are) $x =$ ______

b) The vertical intercept of f is $y =$ ______

c) The local minimum(s) of f is (are) at $x =$ ______

d) The local maximum(s) of f is (are) at $x =$ ______

e) The interval(s) of increase of f is (are) _______

f) The interval(s) of decrease of f is (are) _______

g) Sketch a graph of this function, labeling the axes and the relevant points on the graph.

4) For $f(x) = 2x^3 - 9x^2 + 12x - 3$

   a) Find critical numbers of f.

   b) Find the intervals where f is increasing and where f is decreasing.

   c) Find the local minimums and maximum of f.

   d) Find the intervals of concavity and the inflection points.

   e) Sketch a graph of f using the information above.

5) For $f(x) = \frac{x}{x^2+1}$

   a) Find critical numbers of f.

   b) Find the intervals where f is increasing and where f is decreasing.

   c) Find the local minimums and maximum of f.

   d) Find the intervals of concavity and the inflection points.

   e) Sketch a graph of f using the information above.

6) Find the absolute maximum and minimum values of f on the given interval:

$$f(x) = xe^{\frac{x}{2}}, \qquad [-3,\ 1]$$

7) For the following function describe the domain of the function, the critical numbers on the domain, the intervals where the function is increasing and decreasing and the local minimums and maximums, if any:

$$g(x) = (4 - x^2)^{1/3}$$

8) Find the intervals of concavity and the inflection points for $f(x) = x^4 + 6x^3 + 12x^2$.

9) For the following function describe the domain of the function, the critical numbers, the intervals where the function is increasing and decreasing, and the local minima and maxima, if any:

$$g(x) = (x^2 - 9)^{\frac{1}{3}}$$

10) A manufacturer incurs the following costs in producing x water ski vests in one day, for $0 < x < 20$: fixed costs, $125; unit production cost, $10 per vest; equipment maintenance and repairs, $0.05x^2$ dollars. So, the cost of manufacturing x vests in one given day is given by

$$C(x) = 125 + 10x + 0.05x^2, \text{ where } 0 < x < 200.$$

   a) What is the average cost $\bar{C}(x)$ per vest if x vests are produced in one day?

   b) Find the critical numbers of $\bar{C}(x)$, the intervals on which the average cost per vest is decreasing, the intervals on which the average cost per vest is increasing

   c) Determine the local extrema of $\bar{C}(x)$ and interpret the results.

   d) Determine the intervals of concavity of $\bar{C}(x)$ and points of inflection, if any.

   e) Sketch a graph of $\bar{C}(x)$.

11) A company is using the following model to estimate the number of units $N(x)$ of a product after spending x thousand on advertising:

$$N(x) = -0.25x^4 + 13x^3 - 180x^2 + 10000 \qquad 15 \le x \le 24$$

a) What are the sale trends (when are the sales increasing and when are they decreasing)?

b) Is there a point of diminishing or increasing returns?

c) What is the maximum rate of change of sales?

d) Sketch the graphs of N and N' on the same coordinate system.

12) An outboard motor has an initial price of $1,920. A service contract costs $300 for the first year and increases $120 per year thereafter. The total cost of the outboard motor (in dollars) after n years is given by $C(n) = 30n^2 + 150n + 1920$, where $n \ge 1$.

a) Write an expression for the average cost per year, $\bar{C}(n)$, for n years.

b) Graph the average cost function $\bar{C}(n)$.

c) When is the average cost per year minimum? (This time is frequently referred to as the replacement time for this piece of equipment.)

13) If the revenue function for a certain type of product can be described by

$$R(x) = -1{,}600x^3 + 14{,}400x^2, \qquad 0 \le x \le 9$$

where x (in thousands) represents the demand for the product:

a) When is the revenue increasing and when is it decreasing?

b) Identify the demand for which the revenue is at its maximum.

c) Identify the concavity intervals, the inflection point(s) and interpret the results.

14) Zenus is releasing a new outdoor barbeque model and models its projection of total sales S (in millions of units sold) t years after the sale launch using

$$S(t) = \frac{17.8t}{3.2 + t}$$

Graph the total sales projections for the first 5 years after sales launch, justifying your graph algebraically through the determination of intervals of increase and decrease in total sales, the local extrema, the concavity intervals and the points of inflection, if any.

15) In marketing it is known that different levels of visibility of a certain product in a person's social sphere can increase or decrease the chance the person will make the purchase of the same product. Suppose a team of marketing analysts determined a model, for a certain product, the probability P (in %) that a person will purchase the product depends on the number of people n in their social sphere who already have the product:

$$P(n) = \frac{n}{16 + n^2} + 0.02$$

a) Graph the function $P(n)$ by determining its interval of increase and decrease, local extrema, if any, concavity intervals and points of inflection, if any, and its behavior as n increases towards infinity.

b) Interpret the behavior of the graph in the context of the business model presented.