Section 2.5 Chain Rule

1) Find the derivatives of the following functions.

a) $f(x) = 5e^{-x^2}$

b) $g(x) = (3 + 2x^2)^4$

c) $p(x) = \frac{2}{(3x+1)^2}$

d) $g(x) = \sqrt[3]{1 + 6x^3}$

e) $h(x) = \log_3(-0.2x^3 + 2x^2)$

f) $R(p) = (-0.05p^4 + 5p)^{1/3}$

g) $y = \sqrt[3]{2e^x - 2^x + 1}$

h) $C(x) = \ln(4.4x + 12)$

i) $S(t) = 3(t^{-2} - 1)^{10}$

k) $M(x) = \sqrt{e^{4x}}$

j) $y = 5x - \ln(x + 3)$

l) $p(x) = -\frac{15}{\sqrt[3]{-0.05x^4+3.3x^2-35x}}$

2) The number x of chugpods people are willing to buy per week from a small retail store chain at price \$$p$ per chugpod is given by

$$x(p) = 1208 - 152\sqrt[3]{p + 25} \quad 100 \le p \le 400$$

a) Find dx/dp.

b) Find the demand and the instantaneous rate of change of demand with respect to price when price is \$225. Write a brief interpretation of your results.

3) The total cost (in hundreds of dollars) of producing x calculators per day is given by the equation.

$$C(x) = 9 + \sqrt{2x + 28} \qquad 0 \le x \le 50$$

a) Find the marginal cost function.

b) Determine $C'(18)$ and $C'(36)$ and interpret the results.

4) The price (in dollars) for a weekly demand of x organic peanut butter jars

$$p(x) = \frac{3{,}000}{3x + 50}$$

a) Determine the marginal price function.

b) What is the marginal price if the weekly demand is 150? Interpret the result.

5) Suppose the price-demand function for a product can be modeled by

$$p(x) = 300xe^{-0.05x}$$

where p is price per unit when the demand is x units (in thousands). Find the revenue and the marginal revenue when the demand is 10, 20 and 30 thousand units.

6) Suppose that, based on passed sales of its signature Stocking Stuffer candy mix, the store determined that people are willing to buy x kilograms of the mix per day at \$$p$ per kg following the price-demand equation

$$x = 60 - 20\sqrt{0.5p + 4}, \qquad 1 \le p \le 10$$

a) Find the demand and the instantaneous rate of change of demand when price is \$5/kg.

b) Interpret both results.

7) Suppose that the annual supply of x candleholders (in thousands) at a price $\$p$ is given by

$$x = \sqrt[4]{2.1p - 0.1p^2 + 2p + 7} \qquad 5 \leq p \leq 60$$

a) Find the rate of change in supply with respect to price.

b) Find the supply and the instantaneous rate of change in supply with respect to price when the price is \$40. Write a brief interpretation of these results.

c) Estimate the supply when the price is \$41 using your results in b).

8) Suppose that the annual demand for x lampshades (in thousands) can be sold at a price $\$p$ given by

$$x = \frac{15}{\sqrt[4]{0.05p^4 - 2.2p^2 + 30p}} \qquad 5 \leq p \leq 30$$

a) Find the rate of change in demand with respect to price.

b) Find the demand and the instantaneous rate of change in demand with respect to price when the price is \$20. Write a brief interpretation of these results.

9) Suppose that the price demand function of x units of a product at $\$p$ per unit is given by

$$p(x) = 250\ln(-0.05x + 30)$$

a) Calculate $p'(100)$ and $p'(300)$ and interpret your results.

b) Determine the revenue function $R(x)$ and calculate the marginal revenue at 100 units and 300 units. Interpret your results.

c) Determine the average revenue function and calculate the marginal average revenue at 100 units and 300 units.

10) A marketing company working for a textile manufacturer has developed a marketing campaign for the manufacturer's line of cozy blankets. The company's analysts determined that the projected sales S (in thousands of dollars) will depend on the daily number n of click-throughs on the campaign's social media ads as follows:

$$S(n) = -5.6e^{-0.02n+3.8} + 332$$

a) Calculate $S(50)$ and $S'(50)$ and interpret your results.

b) Calculate $S(100)$ and $S'(100)$ and interpret your results.

c) Graph $S(n)$ using a graphing calculator (for example, Desmos.com/calculator) and explain how the graph of the function relates to your results in a) and b).