Section 2.4 Product and Quotient Rules

1) Find the derivatives of the following functions.

a) $f(x) = 5xe^{-4x}$

b) $g(x) = x(\ln x + 5)$

c) $f(x) = \frac{x^3}{3x+1}$

d) $g(x) = \frac{2x-11}{x^2+3x-4}$

e) $h(x) = \log_3 x(-0.2x^3 + 2x^2)$

f) $R(p) = (-3\sqrt{p^5} + 200p)(1 - e^{-0.02p})$

g) $p(x) = \frac{-2x^{3/2}+54}{x-3}$

h) $C(x) = \frac{200}{x+50}$

i) $S(t) = \frac{3t^2-5t+4}{5e^t}$

j) $M(x) = \frac{\log_6 x}{2x^6-3}$

2) If the average cost per unit $\bar{C}(x)$ to produce x units of sheathing is given by $\bar{C}(x) = \frac{800}{x+25}$, calculate $\bar{C}(20)$ and $\frac{d}{dx}\bar{C}(20)$, and interpret the meaning of the results.

3) Zenus is releasing a new outdoor barbeque model and models its projection of total sales S (in millions of units sold) t years after the sale launch using

$$S(t) = \frac{17.6t}{2.2 + t}$$

a) Find $S'(t)$.

b) Find $S(3)$ and $S'(3)$. Round your answer to three decimals and interpret the results.

c) Use the results in b) to approximate the total number of units sold 4 years after sale launch.

4) The total views (in millions of views) of a movie t months after its online release is

$$V(t) = \frac{35.4t^2}{t^2 + 6.4}$$

a) Find $V'(t)$.

b) Find $V(6)$ and $V'(6)$. Write a brief interpretation of the results.

c) Use the results in b) to approximate the number of views 8 months after release.

5) Suppose the price-demand function for a product can be modeled by

$$p(x) = 300xe^{-0.05x}$$

where p is price per unit when the demand is x units (in thousands). Find the revenue and the marginal revenue when the demand is 10, 20 and 30 thousand units.

6) A new line of carpenter tacks is expected to have its price p relate to the demand of x thousand packages using the following equation:

$$x(p) = 500p^2e^{-0.8p}$$

a) Determine $\frac{dx}{dp}(1)$ and interpret the result.

b) Evaluate $\frac{dx}{dp}(3)$ and interpret the result.

7) Suppose that the weekly supply of x of headphone sets can be sold at a price $\$p$ given by

$$x = \frac{200p}{0.2p+1} \qquad 10 \le p \le 80$$

- **a)** Find the rate of change of supply with respect to price.
- **b)** Find the supply and the instantaneous rate of change of supply with respect to price when the price is $40. Write a brief interpretation of these results.

8) Suppose the profits (in thousands of $) from sales of x thousand units can be determined using

$$P(x) = (292.5x^2 - 945x + 270)\ln x + 42.28$$

- **a)** Determine the profit and the marginal profit at 400 units.
- **b)** Use the results in a) to estimate the profit at 500 units sold.
- **c)** Determine the average profit function $\bar{P}(t)$.
- **d)** Determine the average profit and the marginal average profit at 400 units.
- **e)** Use the results in d) to estimate the average profit at 500 units sold.