

Section 2.3 The Basic Rules for Derivatives

1) Find the derivatives of the following functions.

a) $f(x) = 5x$

b) $g(x) = 5$

c) $f(x) = x^5$

d) $g(x) = x^{1/5}$

e) $R(x) = -0.2x^3 + 2x^2$

f) $R(p) = -2p^4 - 3\sqrt{p^3} + 200p^2$

g) $p(x) = -2x^{5/2} + 534$

h) $p(x) = 2000x^{-1/2} - x^{3/4}$

i) $x(p) = -2000p^{1/4} - p^{4/3} + 7000$

j) $x(p) = \frac{180}{\sqrt[8]{p}} - 4.2\sqrt[5]{p^6} + 791$

k) $S(t) = 135t - 42t^{5/8} + 253.8$

l) $P(b) = 68.4 - 5.2\sqrt[5]{b^3}$

m) $\bar{C}(x) = 150 + \frac{3580}{x}$

q) $\bar{P}(x) = \frac{-0.05x^5 + 3\sqrt[3]{x^4} - 0.6}{x}$

n) $y = e^x + x^e + e$

r) $x = 1000e^{-0.02p}$

o) $S(m) = x^{0.2} - 0.2x + 0.2 \ln x$

s) $p(q) = \log_{0.54} q + 63.1$

p) $A(t) = 3000e^{0.04t}$

t) $m(s) = 5 \log_3 s^4 + (0.8)^{2s}$

2) If the revenue R from an electronics component used in a production of x microchips can be determined by $R(x) = x(125 - 3x)$, determine $R(10)$ and $R'(10)$ and interpret the meaning of the results.

3) Financial analyst at a metal goods manufacturing firm determined the daily cost of manufacturing x units of one of the products is given by $C(x) = 2.4x^{4/5} + 0.3x + 55000$.

a) Calculate the cost and the average cost at daily production level of 2000 and 6000 units.

b) Calculate the marginal cost and the marginal average cost at daily production level of 2000 and 6000 units. Interpret your results.

4) The demand for a product is given by $x = 280 - 3.5p$, where p is the price (in \$) per unit of product and x is the number of units demanded at this price.

a) Write the revenue $R(p)$ as a function of price.

b) Calculate $R'(30)$ and interpret the result.

c) For which values of p is $R'(p)$ positive and for which is it negative? What does the answer tell you about the revenue itself in relation to the price?

5) The total cost (in \$) of manufacturing x units of a product is given by:

$$C(x) = 15,000 + 12x$$

a) Find the average cost per unit if 400 units are produced.

b) Find the marginal average cost at a production level of 400 and interpret your result.

c) Use the results in parts a) and b) to estimate the average cost per unit if 401 units are produced.

6) A marketing analyst working for a sail boat manufacturer has determined that the company can expect to sell $S(x)$ of their Shark sail boats after spending after spending \$ x thousand on advertising, as given by

a) $S(x) = 1590 - \frac{4,380}{x} \quad 5 \leq x \leq 40$

b) Find $S'(x)$.

c) Find $S'(10)$ and $S'(20)$ and interpret the results.

7) A company's monthly sales (in millions of dollars) t months from now are given by

$$S(t) = 0.015t^4 - 0.4t^3 + 3.4t^2 - 10t + 30$$

a) Find $S'(t)$.

b) Determine $S(4)$ and $S'(4)$ (to three decimal places) and interpret your results.

c) Determine $S(7)$ and $S'(7)$ (to three decimal places) and interpret your results.

8) Suppose that at a local bulk food store the daily demand on x kg of party mix priced at p per kg can be modelled by the price-demand equation

$$x(p) = 10 + \frac{180}{p} \quad 2 \leq p \leq 10$$

a) Find $x(6)$ and interpret the result.

b) Find $x'(6)$ and interpret the result.

9) An investment of \$2,000 earns interest at an annual rate of 3.4% compounded continuously.

a) Find the instantaneous rate of change of the amount in the account after 2 years.

b) Find the instantaneous rate of change of the amount in the account at the time the amount is equal to \$2,500.

10) A textile company is releasing a new line of beach towels. Their analysis of sales of models sold in earlier releases, coupled with some new marketing strategy potential, resulted in the following price-demand model between the price p per towel (in dollars) and the demand x (in thousands) for the towels in the new line:

$$p(x) = -0.03x + 0.2x^{\frac{1}{2}} + 14.4 \quad 10 \leq x \leq 650$$

a) Determine the revenue function $R(x)$.

b) Find the marginal revenue function $R'(x)$.

c) Compute $R'(150)$. Round your answer to the nearest integer and briefly explain what the result means in the context of this question.

d) Compute $R'(500)$. Round your answer to the nearest integer and briefly explain what the result means in the context of this question.

11) A company's total sales (in millions of dollars) t months from now are given by

$$S(t) = \sqrt{t} + 2$$

a) Find the average change in sales per month from 25th to 26th month.

b) Find $S'(t)$.

c) Find $S(25)$ and $S'(25)$ and interpret their meaning.

- d) Use the results in c) to estimate the sales 26 months from now. Compare this result with the sales predicted by the sales model function S .

12) The total profit (in \$) from the sale of x electric barbeques is

$$p(x) = 20x - 0.02x^2 - 320 \quad 0 \leq x \leq 1,000$$

- a) Find the average profit per barbeque if 400 grills are produced and sold.
- b) Find the marginal average profit at a sales level of 400 barbeques and interpret the result.
- c) Use the result from a) and b) to estimate the average profit per barbeque and interpret your result.
- d) Use the results from parts a) and b) to estimate the average profit per barbeque if 450 barbeques are sold.
- 13) Using the data from the [2019 Communication Monitoring Report](#), wireless broadband population coverage (%) in Canadian rural communities since 2013 can be modeled by

$$w(t) = -61.836(0.412656)^t + 96.9666$$

where t is years since 2013. Using the model, estimate the Canadian rural community LTE population coverage and the rate of change in the coverage in 2021, and interpret your results.

- 14) At a price of \$2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of \$2.37 per bushel, the supply is 7,900 million bushels and the demand is 7,800 bushels.
- a) Assuming that price and demand are linearly related, determine the price in terms of demand (the price-demand equation).
- b) If $R(x)$ represents the revenue from sales at the demand of x million bushels of barley, determine the revenue and the marginal revenue if 7,600 million bushels are sold.
- c) Assuming that price and supply are linearly related, determine the price in terms of supply (the price-supply equation).
- d) If $C(x)$ represents the cost of purchasing x million bushels of barley from the suppliers, determine the cost and the marginal cost if 7,600 million bushels are to be purchased.

15) Consider the price-demand function and the cost function, respectively, for the manufacture of hammers:

$$p(x) = 15 - 0.03x \text{ and } C(x) = 2x + 700, 0 \leq x \leq 500$$

- a) Find the marginal cost, average cost, and marginal average cost functions.
- b) Express the revenue in terms of x , and find the marginal revenue, the average revenue, and the marginal average revenue functions.
- c) Find the profit, the marginal profit, the average profit, and the marginal average profit functions.
- d) Find the break-even point(s).
- e) Evaluate the marginal profit at $x = 200$, 250, and 350, and interpret the results.

- f) Graph $R(x)$ and $C(x)$ on the same coordinate system, and locate regions of profit and loss. Verify your answers in e) visually.
- 16) A company has developed a new model of a hair iron and has turned to its marketing department for price analysis. After conducting market surveys, the marketing analysts provided the following estimates: at a price of \$16 per iron, the weekly demand would be 250, while at the price of \$14 per iron, the weekly demand would be 300. The operations department puts the fixed weekly costs at \$1600 and the cost per unit at \$4. Find the marginal cost, average cost, and marginal average cost functions.
- a) Assuming that the relationship between demand x and price p is linear, use the marketing department's estimates to write p as a function of x , and determine its domain.
- b) Find the revenue function R in terms of x and its domain.
- c) Assuming that the cost function is linear, using the operations department's estimates to write cost C as a function of x
- d) Graph $R(x)$ and $C(x)$ on the same coordinate system, and locate regions of profit and loss.
- e) Find the profit function $P(x)$.
- f) Evaluate the marginal profit at $x = 150$ and $x = 525$ and interpret the results.
- 17) Assume that \$15,000 was deposited to a savings account earning interest at the rate of 2.19% compounded monthly.
- a) How many months will it take for the amount to double?
- b) What will be the rate of change in the amount in the account when the amount in the account is doubled? Interpret your result.
- 18) An online Mongolian store sells camel wool shawls. Analysing the market and their past sales, the store determined that, if they sell x shawls at a price of $\$p$ per shawl, then the price-demand equation can be expressed as $p(x) = -110 \ln x + 773$. Find the rate of change of price with respect to demand when the demand is 400 shawls and interpret the result.
- 19) A company's financial analysts are projecting the company's profits over the next decade to follow the model
- $$P(t) = 8.25 \log_{2.5} t + 80 \quad 1 \leq t \leq 10$$
- where P is profit (in millions of dollars) and t is years from now. Determine the marginal profit in 3 years and 6 years, and interpret your results.
- 20) Write the function $A(t)$ that describes the return on an investment of \$2300 in t years with interest rate of 1.5% compounded continuously and determine the instantaneous rate of change in the return on the investment after 5 years and after 10 years. Interpret your results.
- 21) Determine the function $I(t)$ that describes the interest in t years owed on a loan in the amount of \$23,000 with interest rate of 2.57% compounded continuously and determine the instantaneous rate of change in the interest after 2 years and after 5 years. Interpret your results.

- 22) A local restaurant estimates that weekly sales s and weekly advertising costs x (both in dollars) are related by

$$s(x) = 14,000 - 13,000e^{-0.0006x}$$

- a) Determine $s(5000)$ and $s'(5000)$ and interpret the results.
 - b) Determine $s(10000)$ and $s'(10000)$ and interpret the results
- 23) A soil decontamination company bidding on a contract with a municipality to clean up an abandoned industrial site estimates that the percentage of soil contamination will decrease by 15% with each \$1000 increase in cost.
- a) Write the function $p(x)$ that describes the percentage of soil contamination (measured in %) at the cost of x thousand dollars if the current, pre-cleanup, contamination is estimated to be 65% and determine what the cost would be to reach the soil contamination level of 10%.
 - b) Calculate $p(10)$ and $p'(10)$, and interpret the results.
 - c) Calculate $p(20)$ and $p'(20)$, and interpret the results.