Section 2.10 Implicit Differentiation and Related Rates

1) In the following problems, use implicit differentiation to determine the given derivative and to find the rate of change at the given point.

   a) $y^3 + x^2 - 12 = 0; \frac{dy}{dx}\ at\ (2,2)$

   b) $y^3 + x^2 - 12 = 0; \frac{dx}{dy}\ at\ (2,2)$

   c) $p^2 + 3p + 2x = 8; \frac{dp}{dx}\ at\ (2,1)$

   d) $p^2 + 3p + 2x = 8; \frac{dx}{dp}\ at\ (1,2)$

   e) $xy^2 = 4 + x; \frac{dy}{dx}\ at\ (2,-2)$

   f) $xy^2 = 4 + x; \frac{dx}{dy}\ at\ (-2,2)$

   g) $2m^3n - m^3 + 5 = 0; \frac{dm}{dn}\ at\ (3,-1)$

   h) $2m^3n - m^3 + 5 = 0; \frac{dn}{dm}\ at\ (-1,3)$

   i) $x^3 + y^3 = e^y; \frac{dy}{dx}\ at\ (1,0)$

   k) $x^3 + y^3 = e^y; \frac{dy}{dx}\ at\ (0,1)$

   j) $3\ln y + x = 2y^2; \frac{dy}{dx}\ at\ (-1,1)$

   l) $(y - 2x)^3 = 2x^2 - 3; \frac{dx}{dy}\ at\ (1,1)$

2) Given the following price-demand equations, find the rate of change in price p with respect to demand x at a given price point and interpret your result.

   a) $x = -p^3 - 3p^2 + 2p + 1516; at\ p = \$2, x = 1500\ units$

   b) $xp^2 + x^3p = 10;\ at\ p = \$1, x = 2\ thousand\ units$

   c) $(x + p)^2 + x = 22 - 3p; at\ p = \$1, x = 3\ thousand\ units$

   d) $x = 1000\sqrt[3]{145 - p^3};\ at\ p = \$4, x = 5000\ units$

3) A sales analyst determined that the monthly sales (in thousands of dollars) and the monthly advertising costs (in dollars) are related by

$$S(a) = 50 - 35e^{-0.0004a}$$

   If the current advertising costs are \$2,500 and the costs are increasing by \$400 per week, find the current rate of change in sales.

4) A sales analyst determined that the monthly sales (in thousands of dollars) and the monthly advertising costs (in dollars) are related by

$$S(a) = 60 - 35e^{-0.0002a}$$

   Find the rate of change in sales when the advertising costs are \$5,500 and the money allocated to the advertising budget is decreasing by \$200 per week.

5) The price p (in dollars) and the demand x (in units) are related by

$$x^2 + 3xp + 25p^2 = 65000$$

   a) If the demand is increasing at the rate of 5 units when there is a demand for 120 units, find the rate of change in price.

   b) If the price is decreasing at a rate of \$3 per month when the price is \$20, find the rate of change in demand.

6) A company's revenue and cost can be modeled by

$$R(x) = -\frac{x^2}{40} + 500x; \quad C(x) = 40x + 80000$$

where the weekly production is x units of a particular product. If the production is increasing by 300 units per week at the production level of 4000 units, find the rate of increase or decrease in

a) Revenue

b) Cost

c) Profit