Section 1.5 Quadratic Functions Exercises

1) Solve the following equations.

   a) $16x^2 - 24x + 9 = 0$

   b) $x^2 = 6x - 13$

   c) $1.5x^2 - 6.3x - 10.1 = 0$

2) For each of the functions below, determine the domain and range of the function, the minimum or maximum value and where it occurs.

   a) $f(x) = 2x^2 + 8$

   b) $g(x) = -500x^2 + 3000x - 5000$

3) Solve the following equations for the given variable:

   a) Solve $5m^2 + 35m - 40 = 0$

   b) Find the roots of $(w - 1)(w - 2) = 6$

   c) Solve $7 + \frac{8}{p} = \frac{12}{p^2}$

   d) Find all real solutions to $\frac{x-12}{3-x} = \frac{x+16}{x+1}$

4) A backyard farmer wants to enclose a rectangular space for a new garden. She has purchased 80 feet of wire fencing to enclose 3 sides, and will put the 4th side against the backyard fence. Find a formula for the area enclosed by the fence if the sides of fencing perpendicular to the existing fence have length L. What dimensions should she make her garden to maximize the enclosed area?

5) A local newspaper currently has 84,000 subscribers, at a quarterly charge of $30. Market research has suggested that if they raised the price to $32, they would lose 5,000 subscribers. Assuming that subscriptions are linearly related to the price, what price should the newspaper charge for a quarterly subscription to maximize their revenue?

6) A company is planning to sell a new smart fitness device. Developing the product will cost $700,000, and each product will cost $30 to manufacture. Market research suggests that if they sell the device for $100, they will be able to sell 30,000 items. For each $10 they lower the price, they estimate they will sell 5,000 more items. Assuming quantity demanded is linearly related to price, determine the price that will maximize profit.

7) The supply for a certain product can be modeled by $p = 3q^2$ and the demand can be modeled by $p = 1620 - 2q^2$, where p is the price in dollars, and q is the quantity in thousands of items. Find the equilibrium price and quantity.

8) Suppose a company has determined the price-demand function for one of its products to be

$$p = 10 - 0.001x \qquad 1 \leq x \leq 10{,}000$$

   a) Write the company's revenue R in terms of number of units x.

   b) If the cost can be calculated using $C = 7{,}000 + 2x$, find the break-even point(s).

   c) Determine the profit P in terms of x and find the number of units to be produced and sold in order to make a profit of $\$5{,}000$.

9) A company keeps records of the price from the sale of x units (in thousands) of a product. It determines that the price-demand function is given by $p(x) = 300 - x$. It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x) = 40x + 1600$.

a) Determine the revenue function and identify its domain.

b) Sketch the graph of the revenue function.

c) When will the company reach maximum revenue? What is the maximum revenue?

d) Find the break-even points for this company. (Round answers to nearest 1000.)

e) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

10) The price-demand equation for a certain product is

$$p = 50 - 0.002x$$

where x is the number of units sold per week and p is the price in dollars at which each one unit is sold. The weekly revenue is given by $R = xp$. What number of units sold produces a weekly revenue of $24,500? What is the price they should charge to achieve that revenue level?

11) Suppose that a small Oshawa pottery factory has a daily fixed cost of $3,600 and can manufacture 75 specialty tea sets per day for a total daily cost of $6,000. Suppose its revenue function in terms of number of tea sets x sold per day can be modelled by

$$R(x) = -x^2 + 250x \text{ for } 0 \le x \le 250$$

a) Determine the cost function assuming the production and the cost are linearly related.

b) Determine the average cost function $\bar{C}(x)$.

c) What does the average cost tend to as the production increases and why does the answer make sense in the context of production costs?

d) Find the profit function and state its domain.

e) Find the break-even point(s) and the maximum profit.

f) Sketch the graph of the profit function.

12) A game developer plans to put a new game on the market. They estimate that they would be able to sell 500,000 games if they price the game at $60 and 750,000 if they price the game at $45. If they assume that the relationship between price and demand is linear:

a) Express price p in terms of demand x and explain why this may be useful to the developer.

b) Find the domain of p and interpret what that means.

c) What price should they charge if they wish to sell 900,000 games?

d) What can they expect the demand to be if they set the price at $70?

e) Express demand x in terms of price p and explain why this may be useful to the developer.

13) Suppose that a plant has a daily fixed cost of \$4,000 and can manufacture 50 guitars per day for a total daily cost of \$6,000. Suppose its revenue function can be modelled by $R(x) = -x^2 + 200x$, where x is the number of guitars sold.

a) Determine the cost function.

b) Find the profit function.

c) Find the break-even point(s).

14) A marketing consultancy is planning to sell a new app for small business that would help the businesses tailor their marketing plans. Developing the app will cost \$450,000, and each app subscription will cost \$150 per year to support. Market research suggests that if they sell the app subscriptions for \$1000 per year, they will be able to sell 2,000 annual subscriptions. For each \$100 reduction in price, they estimate they will sell 500 more annual app subscriptions. Assuming the quantity demanded is linearly related to price:

a) Analyze the revenue:

i. Determine the revenue function in terms of demand and its domain.

ii. Find the number of app subscriptions and the price per subscription that will produce the maximum and the minimum revenue and state the what the maximum and minimum revenue are, if any.

b) Analyze the cost:

i. Determine the cost function in terms of units produced and its domain.

ii. Determine when the cost will be at its maximum and at its minimum and state what the maximum and minimum costs are, if any.

c) Analyze the profit:

i. Determine the profit function in terms of units produced and sold and its domain.

ii. State when the profit will be at its maximum and what the maximum profit is, if any.

iii. Determine the number of annual app subscriptions the company would have to sell and the price they would have to charge to break even.

d) Graph the cost-volume-profit chart (revenue, cost and profit on the same graph) and identify on the chart the break-even points and the areas of profit and loss.

15) A company keeps records of the price from the sale of x units (in thousands) of a product. It determines that the price-demand function is given by $p(x) = 300 - x$. It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x) = 40x + 1600$.

a) Sketch the graph of the revenue function.

b) When will the company reach maximum revenue? What is the maximum revenue?

c) Find the break-even points for this company. (Round answers to nearest 1000.)

d) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

16) An appliance company's marketing research department established that, for their new model of a food processor, the price-demand function is

$$p(x) = -0.05x + 114$$

where p is the price in dollars and x is the demand in thousands of food processors.

a) Determine the domain of p.

b) Find the revenue function in terms of demand and its domain.

c) Sketch the graph of the revenue function.

d) Find the number of units sold that will produce maximum revenue.

e) What is the maximum revenue?