Section 1.1 Functions Exercises

- 1) At a coffee shop, the menu consists of items and their prices. Is price a function of the item? Is the item a function of the price? Justify your answers.
- 2) At Ontario Tech University, the final grade a student earns in the course corresponds to a letter grade (see <u>Ontario Tech Grading</u>). Is the letter grade a function of the percentage a student can obtain in the course? Is the percentage a student can obtain in the course a function of the letter grade? Justify.
- 3) Consider information related to bank accounts.
 - a) Is the account balance a function of the bank account number? Justify.
 - **b)** Is the bank account number a function of the account balance? Justify.
- 4) Introduce function notation of your choice using variable names to represent a function that takes as input the name of a month, and gives as output the number of days in that month. State clearly what each variable represents.
- 5) A function N(y) gives the number of police officers, N, in a town in year y. What does N(2005) = 300 tell us?

b)

6) Which of these tables define a function (if any)? Justify.

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-	•
d	
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Input	Output	
2	1	
5	3	
8	6	

Input	Output
-3	5
0	1
4	5

c)

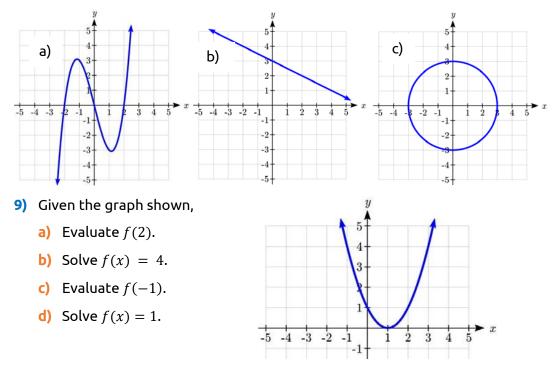
Input	Output	
1	0	
5	2	
5	4	

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7) Using the table shown for Q(n):

n	1	2	3	4	5
Q	8	6	7	6	8

- a) Evaluate Q(3) and interpret your result (describe in words what the answer means).
- b) Solve Q(n) = 6 and interpret your result.
- c) Evaluate Q(7). Interpret your result.
- d) Solve Q(n) = 1. Interpret your result.
- 8) Which of these graphs defines a function? Justify your answers.



- **10)** If possible, express the relationship 2n + 6p = 12 as a function p(n).
- 11) Consider the relationship $3m^2 15n = 7$. Use this relationship to express the following functions, if possible. If this is not possible, justify why not.
 - a) Express n as a function of m.
 - **b)** Express *m* as a function of *n*.

12) Given the function $k(t) = t^3 + 2$.

- a) Evaluate k(2).
- **b)** Solve k(t) = 1.
- **13)** Given the function $h(p) = p^2 + 2p$,
 - a) Evaluate h(4).
 - **b)** Solve h(p) = 3.
- **14)** Given the function $g(m) = \sqrt{m-4}$
 - a) Evaluate g(5)
 - **b)** Solve g(m) = 2
- **15)** A data storage company rents server space for a flat annual fee of \$150 and storage charge of \$0.39 per GB for less than 1.8 petabytes per annual contract (one petabyte is 1000 terabytes and one terabyte is 1000 GB).
 - a) Write the formula for the total cost *C* to rent server space as a function of the gigabytes of data storage requirement *s*.
 - b) What is the domain of this function?
 - c) What is the range of this function?
 - d) Find the total cost to rent server space for 3.5 terabytes of storage space.
 - e) Determine how much server space was rented under the annual contract if the bill was \$1203.
- 16) A rental car company rents cars for a flat fee of \$20 and an hourly charge of \$10.25. Reservations made but cancelled are charged the flat fee. The company policy states that the rentals must be less than 5 days in duration.
 - a) Write the formula for the total cost *C* to rent a car as a function of the hours *h* the car is rented.
 - b) What is the domain of this function?
 - c) What is the range of this function?
 - d) Find the total cost to rent a car for 2 days and 7 hours.
 - e) Determine how long the car was rented if the bill is \$481.25.

17) In making strategic purchasing decisions in business, we study the relationship between the market supply of an item and the price of the item charged by the suppliers. Suppose that for a particular product the number x of units available and the price p per unit demanded by the suppliers satisfy the following relationship:

$$-2x + 35p = 7000$$

- a) If possible, express the supply as a function of the price and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- b) If possible, express the price as a function of the supply and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- 18) In making strategic decisions about product pricing, we study the relationship between the market supply of an item and the price of the item. Suppose the number x of units available and the price p per unit of a particular product satisfy the following relationship: In making strategic decisions about product pricing, we study the relationship between market demand for an item and the price of the item. Suppose the market demand for x number of units and the price of the unit p of a particular product satisfy the following relationship:

$$9x + 150p = 6500$$

- a) If possible, express the demand as a function of the price and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- b) If possible, express the price as a function of the demand and determine its domain. Explain briefly what each of them represent in the context of the question. If it is not possible, explain why.
- 19) If the revenues R from sales of x units of a certain product and the cost C of manufacturing x units of the same product can be expressed as

$$R(x) = 18x$$
 and $C(x) = 9.37x + 25000$

- a) Determine the profit function *P* in terms of the number of units *x*.
- **b)** Calculate *P*(2000) and interpret what that means.
- c) Calculate *P*(4000) and interpret what that means.
- d) How many units would they have to produce and sell to break even?
- e) How many units would they have to produce and sell to earn \$20,000 in profit?
- 20) The population of Oshawa in the year 1960 was 77,000 people. Since then the population has grown to 379,848 people reported during the 2016 census. Choose descriptive variables for your input and output and use interval notation to write the domain and range.

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21) Describe the domains of the following functions using interval notation:

a)
$$f(x) = \frac{3x+5}{x^2+4x-5}$$

b) $f(x) = \frac{\sqrt{2x-3}}{\sqrt{5-x}}$
c) $f(x) = \frac{x}{\sqrt{x+3}}$

- **22)** A phone data plan has a basic charge of \$30 a month. The plan includes first 2GB free and charges \$10 for each additional GB, up to 8 GB total usage, after which it charges \$15 for each additional GB. If *d* is the amount of data used (in GB) and *C* is the total monthly cost:
 - a) Express C(d) as a (piece-wise) formula.
 - **b)** Identify the independent and the dependent variables of *C*.
 - c) Identify the domain and the range of *C*.
 - d) Graph C as a function of d for $0 \le d \le 10$.
 - e) Calculate the cost if 9GB were used.
- 23) The perimeter of a retail store's rectangular parking lot is 200 m. A snow clearance company charges a flat fee of \$350 and \$2.79/m². Determine the cost of the parking lot snow clearance C(w) as a function of the lot width w and state the domain of this function.
- 24) A homeowner has \$540 to spend on building an enclosure around a rectangular garden, which they will build in the corner of their already fenced back yard, using the existing backyard fence for two garden sides. One of two additional sides will be built using wire fencing at cost of \$4 per meter length. The other additional side will be built from wood fencing at a cost of \$12 per meter length and will include a 1.5 m wide gate at a cost of \$150. Write the area of the garden *A* as a function of the length *l* of the wire fencing side.
- **25)** Describe the relationship shown in the graph below as a function and describe its domain and range in interval form.

